

Strategic Withholding and Imprecision in Asset Measurement*

Jeremy Bertomeu

Edwige Cheynel

Davide Cianciaruso

Abstract

How precise should accounting measurements be, if management has discretion to strategically withhold? We examine this question by nesting an optimal persuasion mechanism, which controls what measurements are conducted, within a voluntary disclosure framework à la [Dye \(1985\)](#) and [Jung and Kwon \(1988\)](#). In our setting, information has real effects because the firm uses it to make a continuous operating decision, increasing in the market's belief. Absent frictions other than uncertainty about information endowment, we show that imprecision can reduce strategic withholding but always decreases firm value. We then examine plausible environments under which, by contrast, there is an optimal level of imprecision featuring coarseness at the marginal discloser. We offer additional implications in the contexts of enforcement against strategic withholding and financing with collateralized assets.

Keywords: real effects, imprecision, voluntary disclosure, accounting standards.

*Jeremy Bertomeu and Edwige Cheynel are from the Rady School of Management – University of California San Diego, Wells Fargo Hall, Room 3W124, 9500 Gilman Drive #0553, La Jolla, CA 92093-0553. Davide Cianciaruso is from HEC Paris, 1 Rue de la Liberation, 78350 Jouy-en-Josas Cedex, France. We thank brown bag workshop participants at UCSD and Baruch College. We also gratefully thank Snehal Banerjee, Bradyn Breon-Drish, Gilles Chemla, Ron Dye, Henry Friedman, Pingyang Gao, Denis Gromb, Xu Jiang, Iván Marinovic, Ross Valkanov, Igor Vaysman and Hao Xue for suggestions on earlier versions of the paper.

1 Introduction

In this study, we examine a framework that brings together two classic paradigms in accounting theory: the first paradigm is that of ex-ante design of measurement systems or persuasion, in which a company board or an accounting regulator chooses a measurement that controls what information is collected and reported; the second paradigm is that of strategic voluntary disclosure, such that management may, after receiving unfavorable information, choose to withhold unfavorable news.

Both paradigms are relevant to accounting questions given that the practice of accounting features choices about how to measure transactions and what to report about the measurement. As a continuing motivating example for our analysis, consider the problem of measuring an asset which may have gained or lost value. The firm's accounting standards specify which measurements are conducted during the operating cycle. The process of valuing assets is not straightforward because on occasion, the firm may not always be able to know or show verifiable evidence about changes in value. In the case of a patent, there may not be information about the future cash flows from exploiting the patent; in the case of a property or a private business, no recently traded comparables may be available; in the case of inventories, the firm may be unsure as to whether a decline in demand is permanent or temporary, etc.

Uncertainty about whether the measurement has or has not produced information to be reported in the financial statements creates strategic incentives to withhold information. Specifically, at the end of the accounting cycle, management receives what information has been gathered, if any. Even if evidence has been received that would trigger an impairment test, management may exert discretion not to make this evidence public. Alternatively, we can think about the output of the information gathering stage as information that is initially soft ([Gao and Liang 2018](#)). The accounting system makes some of the information hard by collecting the supporting evidence ([Ijiri 1975](#); [Bertomeu and Marinovic 2015](#)).

Whatever information the measurement produces affects withholding incentives. Understanding this interaction is the objective of this study. Naturally, our model applies to a variety of empirical settings with (a) control over what information is collected, (b) uncertainty about whether

the event has occurred or there is information to be collected. We briefly discuss a few illustrations below - while our model is not intended as descriptive of the institutional details of each of these examples, the settings are meant to offer applications in which the trade-off discussed here would likely be at play.

First, a firm may implement a finer accounting system that brings about information about the occurrence of past misstatements ([Dechow, Ge, Larson and Sloan 2011](#)): for example, this may be achieved in the form of internal controls ([Marinovic 2013](#)) including recurrent reviews and checks on past transactions, hiring more effective auditors or increasing oversight by boards ([Laux 2010](#)). The market does not know whether such evidence has been received, and the manager may decide to conduct the restatement based on the potential impact on earnings or any further reputational consequence. Or, the firm may conduct a restatement stealthily, within venues that make it less forthcoming to market participants ([Files, Swanson and Tse 2009](#)). Management must make a choice whether to report information about a misstatement event to an outside party, but have no simple means to report the absence of the event.

Second, the economic trade-offs may be considered in the context of disclosure of material events, which are in principle required in the US as a filing of form 8-K. In principle, any significant material event should be reported but, in practice, many events may or may not be objectively considered material, and firms can be strategic as to which events to file, or when to file ([Li 2013](#)). The reporting policy of the firm will be a function of how much operational information is collected and transmitted into the financial reporting system. In addition, many material events that could be reported in these filings contain proprietary information, which firms can choose to redact ([Verrecchia and Weber 2006](#); [Heinle, Samuels and Taylor 2018](#)).

Third, the design of the measurement system may interact with accounting measurement, but need not concern only accounting related events. The Foreign Corrupt Practices Act (FCPA) forbids U.S. companies to engage in bribery of foreign officials. Monetary transfers in the process of bribing can, however, be uncovered by accounting internal controls and, therefore, the quality of the accounting system will determine a firm's ability to detect whether its employees violate the

provisions of the FCPA (Cooper, Ho, Hunter and Rodgers 1985). More generally, the quality of the measurement system chosen by the company will allow firms to know whether an illegal act has been committed by an employee. Rogue trading events (Barings, Société Générale), failures to report an environmental violation (Volkswagen) or alleged ignorance of frauds by the Chairman (Enron) are among many examples in which management may have received coarse information from internal measurements.

Within this context, we ask two questions, which we believe to be steps to form a joint theory of optimal measurements and strategic withholding. First, how do we design measurements in the presence of strategic withholding incentives? According to one perspective, we may not need to incorporate strategic considerations into the measurement process if we can rely on voluntary channels to supply information efficiently to the market. Second, how does the existence of the measurement alter the voluntary disclosure environment? We expect very different disclosure behaviors if measurements are precise than if they are imprecise with limited verifiable information. Our purpose is thus to merge the two streams of literature into a theory which speaks about financial reports as a choice of reporting mechanisms constrained by strategic reporting choices.

Before we discuss our main results, we lay out the channels for the interaction between the measurement and voluntary disclosure. Precise measurements can counter-intuitively *increase* incentives to strategically withhold and reduce information. To understand why, suppose that the measurement always obtains precise information about the asset when there exists verifiable information. We know from a vast literature in voluntary disclosure theory that the disclosure threshold will be obtained by comparing (i) the expected value of the asset as assessed by the market when withholding to (ii) the value obtained by the marginal firms in possession of the asset value that choose to disclose that value. By construction, the marginal discloser will be the lowest asset value above the withholding threshold, and thus a precise measurement creates less incentive to disclose.

Suppose that, by contrast, the measurement is imprecise above the disclosure threshold and the information collected by the measurement does not allow the market to know the exact actual

asset value of the marginally disclosed report. Then, the marginal disclosers will compare the non-disclosure price to an expectation over a subset of asset values located above the threshold: this expectation is greater than the threshold signal and, subsequently raises the benefit of disclosure. But there is a downside to this type of measurement: while the set of withheld measurements narrows, the information revealed by marginal disclosers has become more imprecise. In summary, more precision over withholding firms comes at the cost of increasing imprecision over the disclosing firms. This creates a trade-off in choosing the ideal precision of the measurement whose resolution is non-trivial.

We formalize this trade-off theoretically and analyze whether the design of the measurement can improve efficiency in a setting where the firm makes better decisions when public information is more precise. We expect from the trade-off that, in general, the preferred measurement may depend on the value of information, the distribution of the asset values, and the frictions relating to the possible absence of information.

Interestingly, the preferred measurement may collapse to a straightforward measurement if uncertainty about information endowment is the only friction. Within the models of [Dye \(1985\)](#) and by [Jung and Kwon \(1988\)](#), extended to include a choice of measurement and a productive benefit of information, no imprecise measurement would do better than a fully-precise measurement even though imprecise measurements reduce strategic withholding. We delay a complete argument for this result in text and offer here a simple heuristic intuition. The solution to the voluntary disclosure problem prescribes an indifference condition that equates the expectation conditional on non-disclosure to the expectation of the marginal disclosers. When this equality holds, the measurement cannot create dispersion in posterior expectations between withholding firms and marginal disclosers. Without additional dispersion in posterior expectations, a measurement that reclassifies some withholding firms into marginal disclosers creates no useful information.

We relax next a key restriction central to this result and develop a more nuanced answer to the problem. We show, in a framework with both uncertainty about information endowment ([Dye 1985](#); [Jung and Kwon 1988](#)) and *private* costs of disclosure ([Verrecchia 1983](#)), that there is always

some benefit to imprecision for any non-zero disclosure cost or benefit. This framework captures common situations in which information revealed in detailed accounting reports may be used by a different party (a regulatory agency, employees, competitors, consumers, etc.) transferring value from the firm to the other parties. This creates a misalignment between private and social value of information. As management privately bears the cost, the disclosure threshold still equates the payoff from withholding to the payoff from the marginal discloser. In turn, the market expectation conditional on non-disclosure must be lower than the market expectation for the marginal discloser. Therefore, the measurement now generates useful information in the form of different posterior expectations.

Next, we lift the assumption that it would be desirable for the measurement, absent any other friction, to increase the amount of information available. Addressing this question requires additional institutional details about the use of information and we choose here a simple model that captures a possible use of information in debt contracts. Specifically, we embed voluntary disclosure into the collateral financing model of [Goex and Wagenhofer \(2009\)](#), where the firm must meet a minimum asset value in order to finance a project. In this environment, some imprecision for the marginal discloser is desirable as long as the collateral requirement becomes sufficiently large and the probability of receiving information is low. Intuitively, the measurement is designed so that precise measurements are more likely to be voluntarily disclosed over favorable asset values. However, some imprecision for intermediate asset values is used to raise the perceived value of the asset as collateral. Imprecision increases in the probability that information is received.

Literature review. Our study is closely related to two streams of the literature: studies that focus on the structure of the information environment as a determinant of disclosure behavior and studies that focus on the optimal provision of information with real effects.

As to the first stream of literature, the informational environment faced by firms before they release information will affect the type and amount of information that is released. This is a key facet of our analysis and is discussed in papers such as [Einhorn \(2005, 2017\)](#) and [Heinle and Verrecchia \(2015\)](#). [Einhorn \(2005\)](#) considers an environment in which the value of the firm is updated using

two pieces of information, one public and one private, showing that the presence of mandatory disclosures alters the incentives to report news voluntarily. [Heinle and Verrecchia \(2015\)](#) argue that the aggregate expected number of firms making commitments to disclosure affects individual incentives to commit to disclose, implying that the aggregate environment and individual disclosures are jointly determined. The study by [Einhorn \(2017\)](#) is another example in which the existence of a source information competing with voluntary disclosure affects the amount of disclosure.

Within this literature, we are aware of few studies in which the regulatory environment is itself considered as an optimal choice. In order to consider optimal regulatory choice, we thus need to link our paper back to an extensive pre-existing literature on real effects, which (in its most inclusive definition) refers to environments in which information interacts with real operating decisions ([Kanodia 1980](#); [Kanodia and Sapra 2016](#)) and which gives us a rigorous formulation of how properties of a measurement map to a specified objective. This is an active area of literature that is far too broad to make even an attempt toward a comprehensive discussion but some of the recent literature is relevant to our problem.

The paper by [Kanodia, Singh and Spero \(2005\)](#) is most similar to our setting, as it focuses on the optimal choice of imprecision in a noisy signaling problem; a key difference between the context of their paper and our setting is that they do not focus on ex-post incentives to withhold information. In addition, imprecision in their setting takes the form of garbling information while, in our model, it is localized pooling at the lowest disclosure. In [Gao and Zhang \(2018\)](#), the firm makes a commitment to internal control processes which is affected by economy-wide information revealed by other firms. This, in turn, implies that, absent regulation, a firm will under-weigh the value of information and under-invest in its internal controls. The studies by [Goex and Wagenhofer \(2009\)](#), [Lu and Sapra \(2009\)](#), [Beyer and Guttman \(2012\)](#), [Gao and Liang \(2013\)](#) are other recent examples that reveal how information has real effects, although by and large, they focus on either voluntary disclosure or measurement rules but not their interactions.

Our work is part of persuasion theory, which considers the design of a measurement system as a mechanism that prescribes which information is collected and reported. Within this literature, the

studies by [Goex and Wagenhofer \(2009\)](#), [Bertomeu and Cheynel \(2015\)](#), [Huang \(2016\)](#), [Michaeli \(2017\)](#) and [Jiang and Yang \(2017\)](#) bring insights from this literature into the design of measurements that increase efficiency. Within this literature, a few recent studies focus on interaction between persuasion and reporting incentives, including [Friedman, Hughes and Michaeli \(2015\)](#), [Bertomeu, Vaysman and Xue \(2016\)](#) and [Quigley and Walther \(2018\)](#) which develop settings in which public information can crowd out or increase the supply of inside information. A main difference between these models and ours is that, in these models, public information and private disclosures are about different pieces of information. In our model, the persuasion mechanism is the input to the voluntary disclosure.

Two papers are most closely related to ours, in the sense that they focus on persuasion as input to a disclosure decision. [Hummel, Morgan and Stocken \(2016\)](#) focus on a class of problems, in which shareholders can directly set a disclosure policy. With risk-seeking preferences, which closely map to our convex payoff function, they show that the sender prefers full-information. They do not focus on incentives for strategic withholding if the manager may not receive information, which is the main focus of our model. Another related study is by [DeMarzo, Kremer and Skrzypacz \(2017\)](#), which like us, focuses on a persuasion mechanism as input to what is disclosed. They interpret the choice of measurement in terms of an ex-ante test design, but their question is somewhat different. In their model, the agent *privately* chooses a test before being informed and the chosen test is not known conditional on non-disclosure. They show that even without the social value of information, the resulting test is one that meets the minimum principle ([Acharya, DeMarzo and Kremer 2011](#); [Guttman, Kremer and Skrzypacz 2014](#)) that would minimize posterior expectations conditional on non-disclosure.

Two recent papers, while they do not focus on the persuasion mechanism per se, speak to the embedded forces in voluntary disclosure that make the mechanism approach potentially desirable. [Glode, Opp and Zhang \(2018\)](#) examine a sender's disclosure to a monopoly, trading off the gains from trade versus the monopoly's incentive to extract the surplus. As in our model, there are potential benefits from imprecise disclosures. [Rappoport \(2017\)](#) shows that the changes in the

distribution of the information that make the sender less certain about the information translate into lower skepticism by the receiver, and would in many settings increase disclosure. In our model, the mechanism partially controls this skepticism to affect the equilibrium level of disclosure.

2 The model

2.1 Players, Strategies and Payoffs

Our model involves three players: a sender, a receiver, and a designer. For expositional purposes, we use the interpretation of a firm measuring an asset and the state of nature as the asset value. We refer to the designer as the regulator (e.g., a standard-setter), the sender as the manager and the receiver as a capital market forming posterior expectations about the asset value. There is uncertainty about both the asset value and about whether verifiable information regarding the asset exists. The regulator wants to increase communication to the receiver but the sender may strategically withhold to increase the investors' posterior expectation about the asset value.

The timeline is as follows. At time $\tau = 0$, the regulator chooses a measurement system. At $\tau = 1$, the measurement system produces a signal about the asset value whenever it can be measured. The choice of the measurement system is observable to all players. At $\tau = 2$, if verifiable information exists, the manager privately observes the signal and chooses whether to disclose it to investors. At $\tau = 3$, conditional on public information, the investors take an action that affects both the sender and the regulator payoffs. We are interested in the design of a measurement system that maximizes the ex ante payoff of the designer, taking into account the sequentially rational voluntary disclosure decision and market price. Figure 1 depicts the sequence of events.

Distributional assumptions. The firm owns an asset whose value V is random with realizations v in the support $\mathcal{V} \equiv [0, \bar{v}]$, where $\bar{v} > 0$.¹ For instance, one can think about the asset as a retail location, currently owned by the firm, that may serve a role in operations. Let F denote the

¹Throughout, we use a capital letter, V , to denote a random variable and a small letter, v , to denote its realization. Proofs and results apply to distributions with unbounded support.

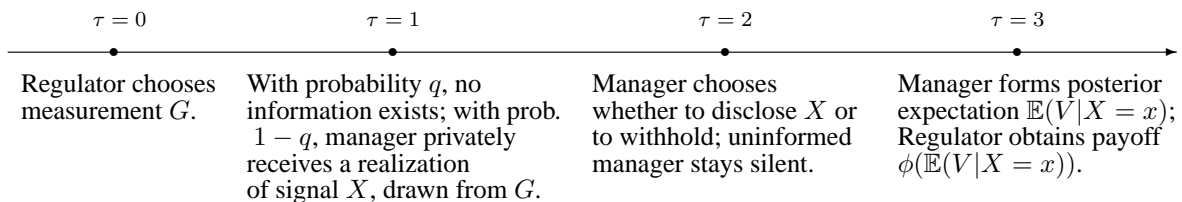


Figure 1: Timeline

distribution function of V . We assume that the distribution of V admits a p.d.f. f that is strictly positive on the support.

As in [Dye \(1985\)](#) and [Jung and Kwon \(1988\)](#), there is uncertainty about whether verifiable information about V exists.² Lack of verifiable information occurs when no information exists or, if it exists, when it cannot be collected and reported credibly (i.e., when information is soft). It may be that, with some probability, there are no comparables to measure the value of a property or a new retail location has not yet generated verifiable long-term sales. We denote by $q \in (0, 1)$ the probability that there does *not* exist verifiable information about the asset value.

The measurement system. As in the persuasion literature ([Aumann and Maschler, 1995](#); [Kamenica and Gentzkow, 2011](#)), we assume that a measurement is a mechanism that controls which information is received by the manager. This measurement could represent prescriptions from accounting rules, internal procedures for collecting information from the field, or long-term accounting choice made by the firm. Given a distribution over states of nature F , a measurement system induces a distribution G over posterior means if and only if F is a mean-preserving spread of G ([Gentzkow and Kamenica, 2016](#)).³ The measurement system chosen thus generates a random variable X , distributed according to G . Because, in our model, only posterior expectations about the asset affect preferences, we can simplify notation by writing each signal X as a posterior

²Note that we expose the model somewhat more generally than [Dye \(1985\)](#), to the extent that our assumption is about the objective presence of (verifiable) information that could be used by the persuasion mechanism. In [Dye \(1985\)](#), by contrast, when information exists about the asset value, it is always verifiable.

³ F is a mean-preserving spread of G if $\int_0^{\bar{v}} \phi(x) dF(x) \geq \int_0^{\bar{v}} \phi(x) dG(x)$ for all convex functions ϕ or, equivalently, if these distributions have equal means and $\int_0^x F(y) dy \geq \int_0^x G(y) dy$ for all $x \in \mathcal{V}$.

expectation about V and restrict G to distributions over the support of X .⁴

When $G = F$, the signal yields the same distribution of posterior means as the asset value itself, that is, the signal X is equal to the asset value V almost surely. This measurement is precise, because it reveals the asset value without noise *whenever verifiable information about it exists*. As to imprecise signals, we do not impose any restriction on the measurement noise. When information does not exist (with probability q), the measurement cannot generate any information, as formalized in the notations below.

Strategy and beliefs. Let Ω denote the random information endowment of the firm. When a firm does not receive verifiable information ($\Omega = \emptyset$), it must choose nondisclosure and cannot convey credibly not to possess any such information. On the other hand, when a firm receives verifiable information $X = x$ ($\Omega = x$), it can report X truthfully or stay silent to pretend that no verifiable information exists, which we refer to as (strategic) withholding.⁵ It could be the case, for example, that the measurement detects that the asset lost value but management passes on a possible impairment arguing to external parties that no such adverse information has been received.

For a given measurement system G selected by the regulator, a strategy for the firm is thus a function $r(\omega, G)$ that maps the firm's private information into a report.⁶ Let r denote the firm's public report (i.e., its disclosure decision), with $r = (d, x)$ standing for disclosure of the posterior mean x and $r = nd$ standing for non-disclosure. When information does not exist, $r = nd$.⁷ A

⁴Specifically, for any signal X , we can use a signal structure that yields the same actions and payoffs $X' = \mathbb{E}(V|X)$. In our model, this signal structure equates the signal to the action taken by the receiver ([Kamenica and Gentzkow 2011](#)).

⁵We have left aside here manipulation incentives to keep our arguments as transparent as possible but, as noted by [Einhorn and Ziv \(2011\)](#), manipulation can be incorporated as an invertible bias into a more general model. An interesting question is whether misreporting costs would create another channel for imprecise measurement to increase value. [Guttman, Kadan and Kandel \(2006\)](#) demonstrate an idea close to this concept by showing that pooling equilibria in misreporting problems can imply higher ex ante value.

⁶We adopt the tie-breaking convention that indifferences between disclosure and nondisclosure are resolved in favor of disclosure. The tie-breaker is inconsequential when the set of marginal disclosers has mass zero. When the marginal disclosers do not have mass zero, this tie-breaker is consistent with the notion that the regulator can slightly perturb the measurement system in such a way as to induce disclosure.

⁷It is critical for the model that the manager cannot, at no cost, gather information that does not exist - or else a fully-precise measurement would be used. As many of our examples show, it is not just plausible, but practically

given disclosure strategy generates a random variable, R , whose realization, $R = r$, is the public report that the market observes.

A market belief is a function $\mu(r; G)$ that maps a report into a posterior expectation about the asset value.

Social value of information. To model the real effects of information, we assume that the firm may make operational decisions as a function of the market's assessment of the asset value, e.g., make investments, draw on external funds, and so forth. We assume that the market values the entire firm at $\phi(\mu(r; G))$, where the function ϕ is increasing, convex, and twice continuously differentiable. This assumption can be rephrased as stating that the ex ante expected value of the firm, $\mathbb{E}(\phi(\mu(R; G)))$ is greater when the market has more precise information in the sense of mean-preserving spreads.

Below, we provide a simple micro foundation that yields conditions such that (i) the maximization of value yields a convex function, and (ii) this function is a function of posterior *expectations*. Suppose that, after the realized report r is observed, the firm makes an investment decision k in order to maximize the market price $\mathbb{E}(Vk - \psi(k)|r)$, where V can be interpreted as productivity and ψ is an investment cost with $\psi(0) = \psi'(0) = 0$ and $\psi'' > 0$ captures decreasing returns to scale.⁸⁹ Differentiating the objective function with respect to investment k to obtain the optimal investment,

$$k^*(\mathbb{E}(V|r)) \equiv (\psi')^{-1}(\mathbb{E}(V|r)). \quad (1)$$

interpretable, that not all information could be gathered and it is a building block of the widely-used [Dye \(1985\)](#) and [Jung and Kwon \(1988\)](#) approaches to model this in terms of a binary event - not for sake of descriptive realism but to convey intuitions. There are cases in which an outside expert would not be able to assess a precise valuation, regardless of how much documentation is provided (consider the problem of valuing an equity stake), or cases where there is evidence (consider valuation with a trade of a comparable asset). Generalizing this model to any possible type of *costly* information acquisition or partial information endowment is not the point of our paper. Further, endogenizing the probability of information endowment with some additional costly process would have little bearing on our main trade-offs.

⁸⁹The claim can be made slightly more general, as we can assume a value of the firm $h(V)\nu(k) - \psi(k)$ and change variables to set $k' = \nu(k)$ and $V' = h(V)$.

⁹⁰Observe that the firm only cares about the market's expectation of the cash flow, and not about the actual cash flow. As a result, there is no mechanism for the firm to signal its private information through the choice of the investment level, because the objective function does not depend directly on the realized X (e.g., see [Brandenburger and Polak, 1996](#)).

Reinjecting this investment into the firm's market price yields a market price that is a function of the posterior expectation about V , that is, letting $x = \mathbb{E}(V|r)$ denote the posterior expectation,

$$\phi(x) \equiv xk^*(x) - \psi(k^*(x)).$$

Differentiating twice the equilibrium firm value $\phi(x)$ yields that $\phi''(x) = 1/\psi''(k^*(x))$. Hence, the convexity of the function $\phi(\cdot)$ directly captures the returns to scale. In the special case of quadratic diseconomies of scale $\psi(k) = k^2$, firm value is quadratic as well, $\phi(x) = x^2/4$, and the ex-ante expectation of firm value is

$$\mathbb{E}(\phi(X)) = \frac{1}{4} [\mathbb{V}_G(X) + \mathbb{E}(V)^2].$$

The ex ante expected firm value in the quadratic cost case thus increases as the informativeness of disclosures increases, where informativeness is measured by the variance of the market's posterior expectation of the asset value.

2.2 Equilibrium

We are now in a position to state our equilibrium concept. The definition is split into two parts. Definition 1 concerns the equilibrium of the voluntary disclosure subgame, after the regulator's choice of the measurement system. Definition 2 describes what constitutes an optimal measurement in this setting.

Definition 1 (Equilibrium of the disclosure subgame) *For a given measurement system G , an equilibrium of the voluntary disclosure subgame is given by a reporting function, $r(\omega; G)$, and a market's assessment of the asset value, $\mu(r; G)$, such that:*

- (i) *Given the firm's reporting function $r(\omega; G)$, the market's assessment $\mu(r; G)$ is the posterior*

expectation of the asset value given Bayes' rule, that is,

$$\mu(r; G) = \mathbb{E}(V|R = r);$$

(ii) Given the market's assessment $\mu(r; G)$, the firm's reporting function maximizes the firm's payoff, that is,

$$r(\omega; G) = \begin{cases} nd & \text{if } \omega = \emptyset \\ (d, x) & \text{if } \omega = x \text{ and } \phi(x) \geq \phi(\mu_{nd}) , \\ nd & \text{if } \omega = x \text{ and } \phi(x) < \phi(\mu_{nd}) \end{cases}$$

where $\mu_{nd} = \mathbb{E}(V|R = nd)$.

This definition is standard: the market posterior expectation must follow Bayes rule (i) and the manager discloses only when informed and when the price is lower conditional on non-disclosure.

Since the measurement system affects the disclosure strategy and the ex ante expected firm value only through the distribution of posterior means that it induces, G , the regulator directly maximizes over measurable signal distributions.

Definition 2 (Optimal measurement) *An equilibrium measurement G^* is optimal if it solves*

$$\max_G \mathbb{E}(\phi(\mu(r(\Omega; G))))$$

s.t. F is a mean-preserving spread of G ,

where $r(\omega; G)$ and $\mu(r; G)$ are equilibrium strategies for the firm and the market, respectively, in the voluntary disclosure subgame.

We say that a measurement is optimal if it maximizes the ex-ante value of the firm. Note that because prices $\phi(\mu(r(\Omega; G)))$ are convex in the posterior expectation $\mu(r(\Omega; G))$, we know that the regulator would always prefer a fully precise measurement if all information could be disclosed, that is, if $q = 0$. In this case, the conditions for unravelling hold ([Viscusi 1978](#)) and the manager's

report is fully informative about the outcome of the measurement. If $q > 0$, strategic withholding will occur on the equilibrium path, implying a control over the measurement is not equivalent to control over the voluntary signal that the market receives.

To keep the exposition simple, we will develop the analysis in-text in the special case of “interval” measurements, which we define precisely in the Definition that follows (Definition 3) but first describe it intuitively. In Appendix B, we show that interval measurements are optimal within the broad class of measurements in Definition 2 but the proof is long and technical and adds limited economic intuition.

An interval measurement is such that the information received by the manager is either in the form of an interval with the form $[a_i, a_{i+1})$, or reveals full information about the true state in the form of a full-disclosure region \mathcal{V}_{FI} . We further economize notation by restricting the attention to measurements in which the full-information region is either empty or a single interval, $\mathcal{V}_{FI} \equiv [a_j, a_{j+1})$, and without loss of generality set the lowest interval $[0, a_1)$ equal to the withholding region.¹⁰ An interval measurement is fully described by $\mathcal{M} = (\mathcal{V}_{FI}, (a_i)_{i=0}^I)$, with a specification of the (possibly-empty) fully-revealing region \mathcal{V}_{FI} and the set of interval information sets $[a_i, a_{i+1}) \neq \mathcal{V}_{FI}$ associated to the measurement.

Definition 3 (Convex-partitional measurement) *A measurement G is convex-partitional if there exists $\mathcal{M} = (\mathcal{V}_{FI}, (a_i)_{i=0}^I)$, with $\{a_i\}$ an increasing sequence from $a_0 = 0$ to $a_I = \bar{v}$, such that either (i) $X = \mathbb{E}(v|v \in [a_i, a_{i+1}))$ for all $v \in [a_i, a_{i+1})$ or (ii) $X = v$ for all $v \in \mathcal{V}_{FI} \equiv [a_j, a_{j+1})$.*

2.3 Discussion

Several assumptions, which we discuss at greater length below, are critical for our model.

Ex-ante measurement system design. As in most of the persuasion literature, a measurement in our model describes which information is collected by an accounting system whose characteristics are known to outsiders (e.g., long-term policies, accounting standards, etc.). The collection of

¹⁰This is without loss of generality as also shown in part of our main proof of optimality in Appendix B.

information begins prior to economic uncertainty being realized, typically during the operating cycle and, if management were to receive additional private information at the end of a reporting period, this information would be soft and no longer be credibly reportable. The model may, therefore, not be a fit for activities that occur *exclusively* at the end of a reporting period, especially in environments where management chooses a measurement with private information about what the measurement would deliver. In our model, it is the regulator who chooses a measurement and does so without knowing the value of the asset and incentives to increase ex-post market values. By contrast, with purely ex-post choice over the measurement system, the unravelling principle would hold and management would face skeptical expectations when choosing anything but a fully-informative measurement.¹¹

Uncertainty about existence of verifiable information. The measurement in our model is constrained by the existence of verifiable information. In particular, even a fully-precise measurement will fail to deliver a reportable signal if verifiable information does not exist. This assumption broadly follows the framework of [Ijiri \(1975\)](#) which views the accounting process as making certain pieces of information hard but notes that not all information can be so incorporated into the accounting process. We give various various examples in the introduction of soft information that could not easily be documented to yield verifiable measurements.

As such, a key assumption in our study is that information can take two forms: information that cannot be measured (entirely soft or subjective) and information that can be measured, e.g., for which there is collectable or verifiable evidence. The occurrence of the friction in our model *and* the chosen measurement *jointly* determine the information endowment of the manager. For obvious reasons (both conceptual and practical), we assume that a measurement cannot collect verifiable information when it does not exist. Of note, the subset of results pertaining to optimal imprecision in Sections [3.2](#) and [4.1](#) (with privately-borne disclosure or withholding costs) apply

¹¹Note that we are not the only study focusing on measurement system design and there is a very extensive prior literature in which measurements are chosen prior to receipt of private information, see [Arya, Glover and Sivaramakrishnan \(1997\)](#), [Kanodia, Singh and Spero \(2005\)](#) or [Gigler, Kanodia, Sapiro and Venugopalan \(2014\)](#), among many other examples.

even if we assume that measurable evidence always exists ($q = 0$).

Social value of information. We assume that there is a social value of information, which we model as a convex function of posterior expectation $\phi(\cdot)$. This formulation leaves aside benefits from imprecision discussed in prior literature, such as inducing more efficient pre-disclosure actions (see, e.g., [Kanodia 2006](#)), since the benefits of imprecision in these settings are the object of an extensive ongoing literature - in this respect, our study is meant to be incremental to other rationales for imprecision covered in this literature (whose main point is not about incentives to disclose ex-post). This is also a manner to take a simple conceptual perspective in which the regulator's problem can be thought as either a welfare problem or, more simply, as increasing an information flow to the market. We discuss in [Section 4.2](#) an extension of our results in the context of the collateral financing problem of [Goex and Wagenhofer \(2009\)](#).

3 Analysis

3.1 Effect of imprecision on disclosure

As a benchmark, we state the equilibrium with perfect measurement. The equilibrium is standard in the literature and such that informed managers disclose information if and only if they observe a state x above a threshold t_0 . We further know that this threshold must satisfy the indifference condition for the marginal discloser $\phi(\mu_{nd}^{t_0}) = \phi(t_0)$, where $\mu_{nd}^{t_0} \equiv \mu(nd; F)$ is the non-disclosure expectation with a perfect measurement. Simplifying, the above equation is simply $\mu_{nd}^{t_0} = t_0$ so that this indifference condition is not a function of $\phi(\cdot)$ and the disclosure threshold is given by equation (7) in [Jung and Kwon \(1988\)](#). Note that, while the social value of information does not affect the localization of the disclosure threshold under perfect measurement, non-disclosure decreases firm value more when $\phi(\cdot)$ is highly convex.¹² Put differently, there is no adjustment in

¹²In the equilibrium of the voluntary disclosure model, the firm value is $(q + (1 - q)F(t_0))\phi(\mu_{nd}^{t_0}) + (1 - q) \int_{t_0}^{\bar{v}} \phi(x)f(x)dx$, versus $q\phi(\mathbb{E}(V)) + (1 - q) \int \phi(x)f(x)dx$ if the manager did not withhold strategically. The loss of expected surplus due to strategic withholding is a function of $\phi(\cdot)$ (and is zero if $\phi(\cdot)$ is linear). However, as noted

the probability of disclosure because the manager only cares about the posterior expectation and not the ex-ante surplus.

To obtain intuition for the benefit of imprecision, suppose that the marginal discloser does *not* receive a perfectly precise signal, but instead observes that $V \in [t, a_2)$. In other words, the expectation at the marginal discloser is $\mathbb{E}(V|V \in [t, a_2))$. This marginal discloser has an incentive to disclose as long as the posterior expectation is greater under disclosure than under withholding:

$$\mathbb{E}(V|V \in [t, a_2)) \geq \mu_{nd}^t, \quad (2)$$

where the non-disclosure posterior $\mu_{nd}^t \equiv \mu(nd; G)$ is simply the expectation of V conditional on either the manager being uninformed or the manager being informed and withholding when $V < t$.

Note a key difference with standard voluntary disclosure theory (Verrecchia 1983; Dye 1985; Jung and Kwon 1988; Acharya, DeMarzo and Kremer 2011; Guttman, Kremer and Skrzypacz 2014). The expectation in equation (2) implies that the marginal discloser receives $\mathbb{E}(V|V \in [t, a_2))$ instead of t under a perfect measurement. Consequently, the manager receives a higher price when disclosing and a smaller withholding region can be sustained in equilibrium.

To illustrate this point further, consider a straightforward binary measurement: the firm knows only whether the asset value is above or below a threshold t (i.e., set $a_2 = \bar{v}$). The equilibrium is always such that measurements with $v \in [0, t)$ are withheld and $v \in [t, \bar{v})$ are disclosed, because these are the lowest and higher payoff, respectively, that can be achieved by the manager. Hence, the regulator can implement any disclosure threshold t , including thresholds below t_0 . But the reduction in withholding carries an informational loss: the increase in information available over low asset values $v \leq t$ translates into a decrease in the information for $v > t$. As the threshold $t \rightarrow 0$, the probability of strategic withholding converges to zero and nondisclosure becomes a perfect signal about the uninformed firm. But, then, the information about all other asset values becomes completely imprecise. Even if the convexity of the payoff function $\phi(\cdot)$ implies some

in text, the voluntary disclosure threshold t_0 and the probability of disclosure $(1 - q)(1 - F(t_0))$ are not a function of $\phi(\cdot)$.

benefits from information, determining which of these two competing effects dominates requires further analysis.

We move next to the characterization of the optimal measurement. We make first a basic observation that greatly simplifies the problem. Recall that measurements with $v \geq a_2$ imply posterior expectations that are strictly greater than the non-disclosure price, so that incentives to withhold are not a binding constraint. Hence, we can focus on implementing the measurement system that maximizes ex-ante value ignoring strategic reporting for asset values above a_2 . From a direct application of Jensen's inequality, the preferred measurement is one that is fully-informative for any $v \geq a_2$.

Lemma 1 *Let \mathcal{M} characterize an optimal measurement with $I \geq 3$, then $\mathcal{V}_{FI} = [a_2, \bar{v})$.*

Lemma 1 does not apply to the region of the minimal disclosed measurement, i.e., $[t, a_2)$ because, in this region, increasing precision further changes the posterior expectation of the marginal disclosers $\mathbb{E}(V|V \in [t, a_2))$ and alters the withholding region.

To engage the next step of the proof, observe that we would ideally want to reduce a_2 as much as possible if we were to ignore the voluntary disclosure problem. To see why, recall from lemma 1 that any v above a_2 is perfectly reported so the informational loss decreases when the information $[t, a_2)$ becomes more precise. Naturally, decreasing a_2 makes the marginal disclosers more willing to withhold so that it can only be continued until the voluntary disclosure problem binds the indifference condition of the marginal discloser $\mu_{nd}^t = \mathbb{E}(V|V \in [t, a_2))$.

Lemma 2 *Let \mathcal{M} characterize an optimal measurement with $I \geq 3$, then $\mu_{nd}^t = \mathbb{E}(V|V \in [t, a_2))$.*

Lemma 2 implies a simple graphical representation of the measurement design problem. We plot in Figure 2 the non-disclosure market value $\phi(\mu_{nd}^t)$ for different thresholds t . We know from Acharya, DeMarzo and Kremer (2011) that the particular disclosure threshold t_0 minimizes the non-disclosure price. Now, consider implementing $t < t_0$, which requires designing an imprecise measurement $[t, a_2)$ that satisfies $\mu_{nd}^t = \mathbb{E}(V|V \in [t, a_2))$.

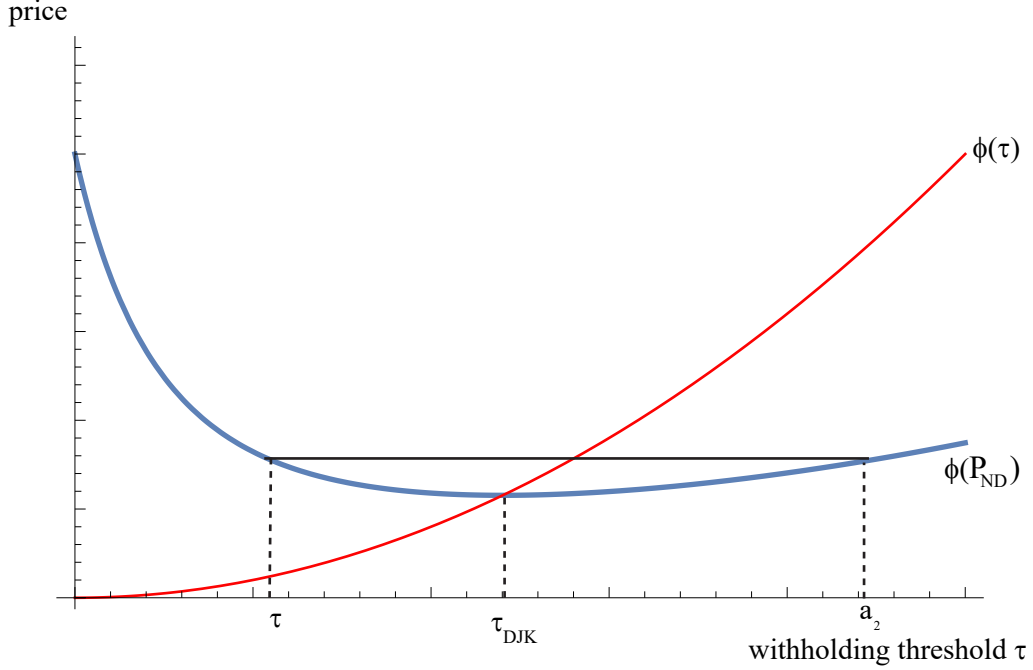


Figure 2: Imprecision at the threshold

The required imprecision $[t, a_2)$ can be obtained graphically. Let us draw a horizontal line intersecting at $(t, \phi(t))$. This horizontal line intersects the non-disclosure price at another point: if the disclosure threshold had been set at this point, the non-disclosure price would be equal to the equilibrium non-disclosure price. It so happens that this point is the desired a_2 . To see why, remark that $\mu_{nd}^{a_2}$ can be decomposed as a weighted average of μ_{nd}^t and $\mathbb{E}(V|V \in [t, a_2))$. Having constructed a_2 so that $\mu_{nd}^t = \mu_{nd}^{a_2}$, it must be that $\mathbb{E}(V|V \in [t, a_2)) = \mu_{nd}^t$ which is what defines the optimal coarse region $[t, a_2)$. To summarize, sustaining a disclosure threshold $t < t_0$ requires an imprecise measurement $[t, a_2)$ delimited by the horizontal line.

When t is close to t_0 , this region converges to a point and no imprecision is required. When t converges to zero, the solution converges to $a_2 = \bar{v}$ and prescribes complete imprecision when the firm is informed. In-between, the region of imprecision must include t_0 so that shrinking strategic withholding would always come with a loss of information over some asset values that would have been disclosed under a fully precise measurement. In fact, while a lower threshold t implies a higher non-disclosure price, we can also observe that the horizontal line at $\phi(\mu_{nd}^t)$ will

intersect $\phi(t)$ somewhere in $[t, a_2)$: despite the potential gain from more information, some firms must achieve a lower price in an equilibrium with imprecision relative to the equilibrium in which $t = t_0$.

While imprecision may hurt some firms and benefit others once the asset value realizes, can it increase expected firm value? To answer this question, Figure 2 has another critical implication. Regardless of how we set the disclosure threshold t , the posterior expectation implied by all asset values less than a_2 must be pinned down by the horizontal line. So, from the point of view of posterior expectations, any measurement with imprecision is equivalent to a simpler measurement in which we set the withholding threshold at a_2 .

Lemma 3 *Let \mathcal{M} characterize an optimal measurement, then $\mathcal{V}_{FI} = [t, \bar{v})$.*

The lemma relies on the property of marginal disclosers in voluntary disclosure equilibria. The strategic withholding condition in (2) rules out any dispersion in posterior expectations near the disclosure threshold. So, while the measurement can elicit more information over unfavorable events, no *useful* information is given that would cause revisions in posterior expectations. Hence, the optimal measurement takes the form of a withholding threshold above which the measurement is fully-informative. This implies that no imprecision is effectively used to discipline more voluntary disclosure than what would emerge in Jung and Kwon (1988).

Proposition 1 *A fully precise measurement is optimal. In this measurement, the asset value is reported if and only if the verifiable information exists and v is greater than the Jung and Kwon (1988) threshold t_0 .*

While frictions may prevent voluntary disclosures from unravelling to reveal all information, it achieves the most useful amount of information for production purposes. Surprisingly, while the measurement can interplay with voluntary disclosure and affect the provision of information about good or bad news, it can never do so in a way that would increase the expected value of the firm. Put differently, *the cost of overcoming strategic withholding is always greater than its benefits on*

reducing withholding.

This may seem surprising given our earlier observations about withholding behavior around the disclosure threshold. The result stands on a simple intuition which can be constructed starting from the minimum principle in disclosure theory (Acharya, DeMarzo and Kremer 2011; Guttman, Kremer and Skrzypacz 2014; Dye and Hughes 2018). The minimum principle implies that the full-information threshold minimizes the withholding price among all other possible thresholds; so, it serves to discipline as much disclosure as would be possible if we could set the threshold at another location. But intuitively, this is the task that the measurement sets out to do, by changing the nature of the information received.

3.2 Private costs of disclosure

We extend the model to a second friction affecting disclosure choices, by assuming as in Einhorn and Ziv (2008) that disclosure may involve both uncertainty about information endowment and private costs. Specifically, the firm internalizes a cost $c > 0$ when making a disclosure. We assume that, plausibly, this cost mainly reflects distributional effects to other parties affected by the disclosure (competitors, consumers, other firms, etc.) and is not viewed by the regulator as a social cost.

We plot in Figure 3 the payoff to a withholding manager as a function of the lower bound t in the imprecision interval, with and without disclosure costs. The solution to Jung and Kwon (1988) with disclosure costs is now located at $t_c > t_0$. Note that imprecision creates dispersion in posterior expectations (a pre-condition for the information to be useful in our model) because the non-disclosure posterior expectation is strictly lower than the posterior expectation of the marginal discloser $E(V|V \in [t, a_2])$. We should try to set t as small as possible but large enough so that marginal disclosers do not deviate to withhold and are compensated for incurring a disclosure cost. The optimal withholding threshold t^* is located between t_0 and t_c , the disclosure in Jung and Kwon (1988) without and with costs, respectively.

Our next result, Proposition 2 below, states that the optimal measurement system always in-

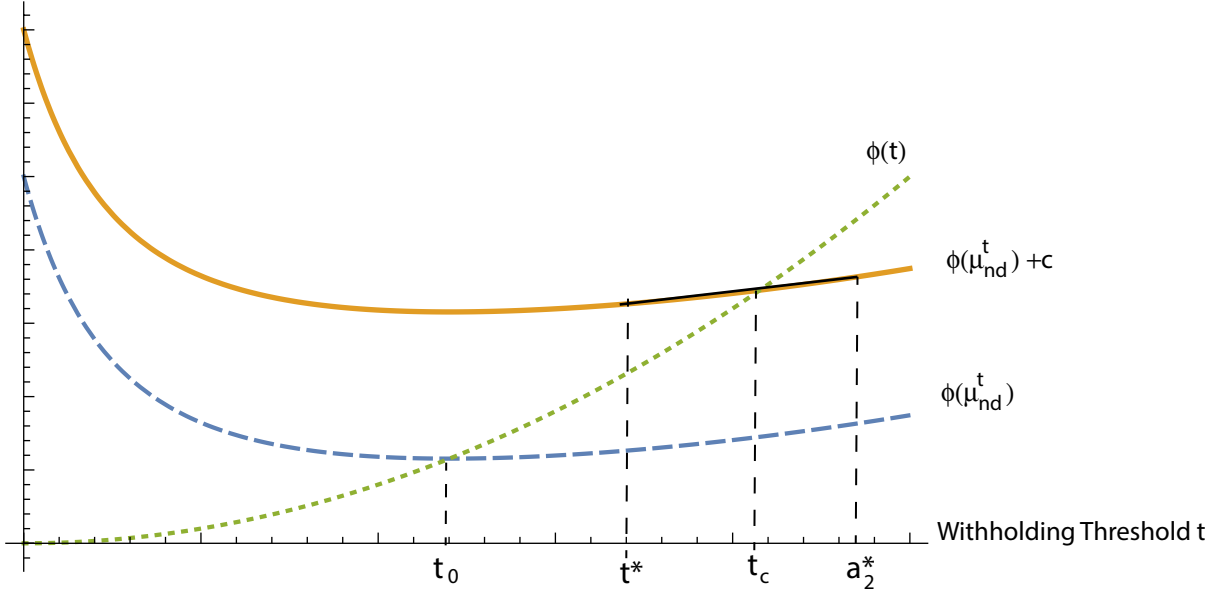


Figure 3: Imprecision with disclosure costs

volves imprecision when disclosure is privately costly but not a social cost.

Proposition 2 *If disclosure is costly (i.e., $c > 0$), the optimal measurement G^* is such that there exists a single non-empty imprecise region $[t^*, a_2^*)$ such that: (i) any state below t^* is withheld, (ii) states in $[t^*, a_2^*)$ are reported coarsely and (iii) any state above a_2^* are reported precisely.*

Let us explain the intuition for this finding in several steps, starting from an intuitive, but incomplete, argument why imprecision becomes optimal with private costs. It is true that private costs cause a misalignment between the firm and the regulator, as the firm internalizes disclosure costs while the regulator does not. However, the disclosure strategy, even absent costs, is not a function of the social value of information $\phi(\cdot)$ and thus already exhibits misalignment between the ex-ante problem solved by the regulator and the ex-post disclosure problem solved by the firm. This translates into a voluntary disclosure equilibrium that yields less disclosure than would be ex-ante desirable to the regulator and, yet, does not require imprecision in Proposition 1. In short, misalignment of objectives is not a sufficient condition for imprecision to be optimal.

The effect of costs here is slightly different. The key to our earlier result is that the regulator

cannot create dispersion in posterior beliefs between a non-disclosing firm and a marginal discloser. With private costs, however, there is additional dispersion in posterior beliefs because the marginal discloser must achieve a strictly greater expectation about the asset value V (gross of cost). Exploiting this, imprecision increases the variation in posterior beliefs around the disclosure threshold. Put differently, disclosure costs separate expectations about fundamentals between non-disclosers and marginal disclosers: an imprecise measurement, by increasing the probability that a firm is a marginal discloser, translates into more variation in posterior beliefs.¹³

We illustrate these results using an example that serves to reveal the expected loss suffered by moving the withholding threshold below t_0 . We start with the model without costs. Let us set V to be uniform on $[0, 1]$ and a market value $\phi(x) = x^2$. If we set a fully-informative measurement \mathcal{M}^{all} , the manager will withhold information (after some cumbersome but otherwise uninteresting algebra) when $v \leq t_0$ is given by $t_0 = \mu_{nd}^{t_0} = 1 - \frac{1}{1+\sqrt{q}}$. Reinjecting this threshold yields an expected value

$$\sigma^{all} \equiv \mathbb{E}(\phi(\mathbb{E}(V|R))) = \frac{1 + 2\sqrt{q}}{3(1 + \sqrt{q})^2}, \quad (3)$$

which, as expected, decreases as the friction q increases.¹⁴

Alternatively, consider measurements in which we elicit more precise disclosures for V below

¹³Another way to understand the result that imprecision is optimal with costly disclosure is as follows. As we depart from perfect measurement and we decrease the threshold t below t_c , two competing forces arise: a *dispersion* effect and an *information loss* effect. The dispersion effect captures the fact that, as we decrease the threshold, we increase the dispersion between the expectation at the marginal discloser, $\mathbb{E}(V|t < V < a_2)$, and the nondisclosure expectation, μ_{nd}^t . Intuitively, the spread between the expectation at the marginal discloser and the nondisclosure price is what compensates the marginal discloser for incurring the disclosure cost. When, by decreasing the threshold, we make the market more skeptic towards nondisclosure, the marginal discloser has to be compensated more, as at lower payoff levels the marginal benefit of a higher posterior expectation relative to the nondisclosure expectation is smaller. Thus, the spread between these two expectations must increase. All else equal, the dispersion effect increases the ex ante expected firm value, since firm value is convex in the posterior expectation. By contrast, the information loss effect represents the fact that the information conveyed by the marginal discloser is now coarser, as the marginal discloser is not a single point t_c anymore, but rather an interval of values, $[t, a_2)$. Hence, all else equal the information loss effect decreases the ex ante expected firm value. In a neighborhood of $t = t_c$, the information loss effect is negligible, because it negatively affects firm value only conditional on the asset being equal to the threshold t_c , which is a zero probability event. As a consequence, the positive dispersion effects dominates the negative information loss effect, and some imprecision is optimal.

¹⁴To derive these expressions, note that the threshold equation from Jung and Kwon (1988) must satisfy

$$(1 - q)\frac{t_0^2}{2} - q\left(\frac{1}{2} - t_0\right),$$

t_0 . Specifically, set $\mathcal{V}_{FI} = [a_2, \bar{v})$ and a coarse region $[t, a_2)$. We vary t and set the minimum a_2 that satisfies (2). If $t \geq t_0$, we can simply set $a_2 = t$.¹⁵ If $t < t_0$, we need to set the expectation in the disclosure region $[t, a_2)$ greater than the expectation when withholding, requiring¹⁶

$$a_2 = \frac{q(1-t)}{q(1-t)+t} > t_0. \quad (4)$$

As observed earlier, the lower the threshold t , the greater the threshold a_2 . Similarly, the greater the disclosure friction q , the more the measurement must increase a_2 . In Figure 4, we plot the resulting expected value for various levels of the friction q . As shown earlier, for each of these plots, the expected surplus peaks at t_0 and there is a loss of useful information for thresholds above or below t_0 .

Assume next that firms bear a private disclosure cost $c > 0$. We plot in Figure 5 several examples following the uniform specification and quadratic payoff $\phi(\cdot)$ used in the previous section, a probability of being uninformed $q = 0.1$ and various choices for c from 0 to 0.3. At $c = 0$, the expected firm value is maximized at $a = t_0$ which means that there is no value in imprecision. As c increases, the total value no longer peaks at t_c and there is an interior imprecise interval. Note that the region of imprecision shifts to the right and shrinks as c becomes small because then withholding becomes increasingly unlikely.

and we can then obtain σ^{all} from

$$\begin{aligned} \sigma^{all} &= (q + (1-q)t_0)(\mu_{nd}^t)^2 + (1-q)(1-t_0)\mathbb{E}(V^2|V \geq t_0) \\ &= \frac{1 + 2\sqrt{q}}{3(1 + \sqrt{q})^2}. \end{aligned}$$

¹⁵This is an innocuous abuse of language since we would have here that $\mathcal{V}_{FI} = [t, \bar{v})$ with no need to coarsen the information above the threshold.

¹⁶This follows from

$$\frac{t + a_2}{2} = \frac{q \times \frac{1}{2} + (1-q)t \times \frac{t}{2}}{q + (1-q)t}.$$

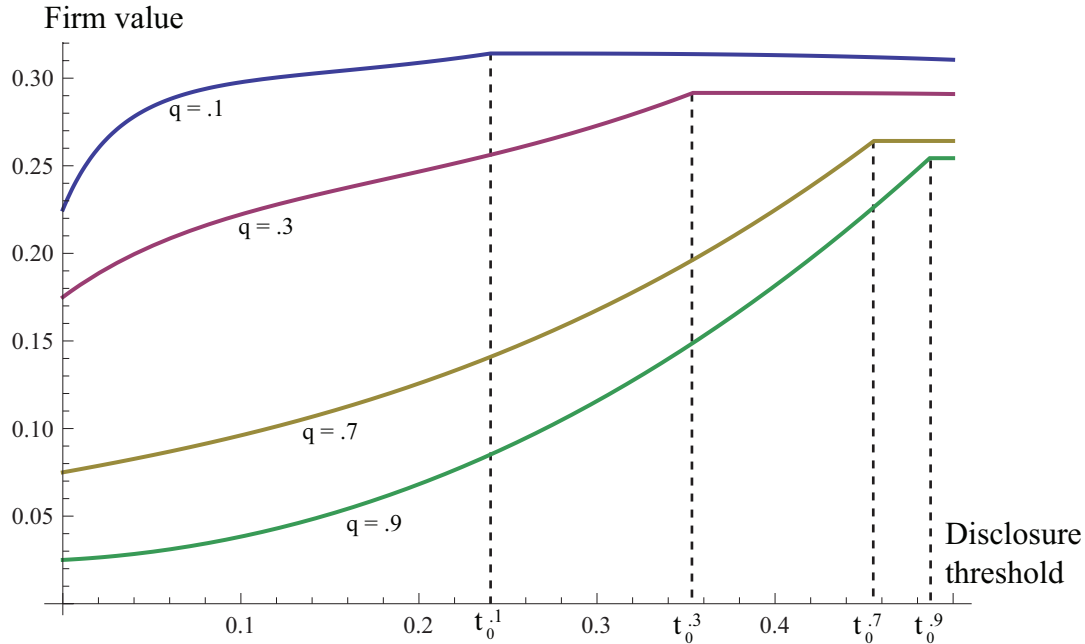


Figure 4: Expected market value from various threshold choices t

4 Extensions

4.1 Withholding Penalties

Although enforcement agencies may not know right away whether a manager chose to withhold information, there may be situations where a regulator and/or court of law may act to enforce against *strategic* withholding at a later date after the fact. For example, in the absence of an impairment, it may come to light that the manager was informed because of an insider leak or some later information emerges that reveals a fraud (e.g., shredded audit documents). These aspects echo various formalizations of this problem where an outside party may reveal whether the manager was informed; see [Dye \(2018\)](#) for a recent analysis.

Because our main focus here is on the choice of measurement, which on its own presents a non-trivial optimal choice, we simplify this problem to capture the first-order effect of individual legal risks in reduced-form. There is a cost borne by the firm management when strategically withholding. We have in mind that the existence of material information is revealed at some date

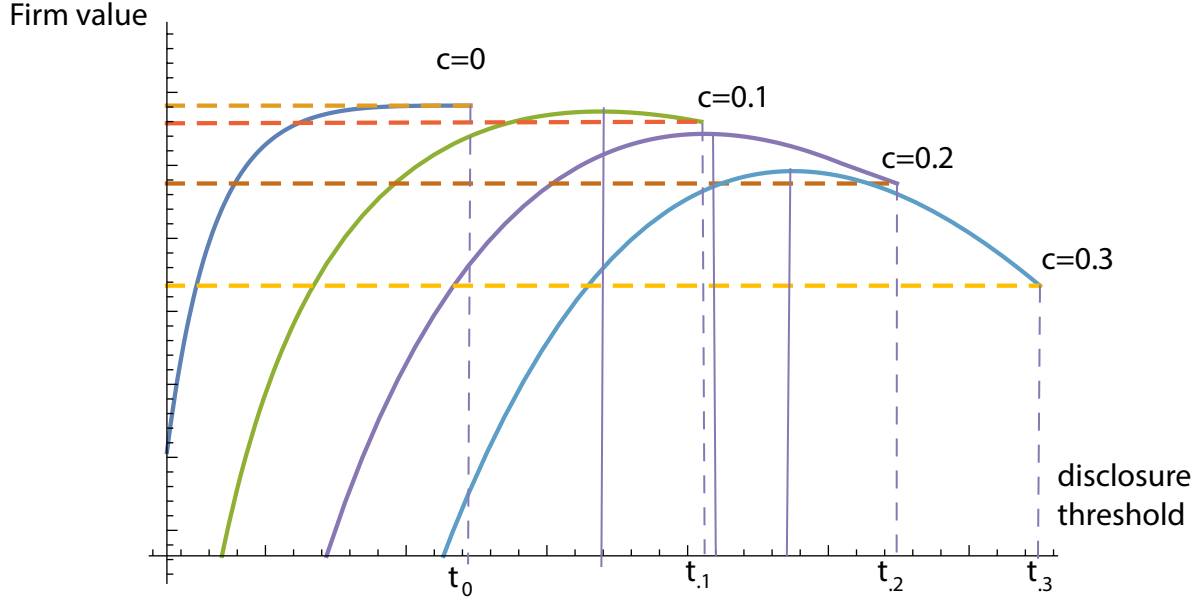


Figure 5: Expected firm value as a function of imprecision ($q = .1$)

in the future, or randomly after the manager makes the reporting choice. More generally speaking, the model speaks to plausible setting in which a manager willingly withholding information would not receive exactly the same payoff as a manager who was uninformed and did not withhold intentionally.

We assume that the signal is about the existence of information and, for parsimony, does not depend on the realized signal. This assumption is made primarily to focus our attention and is not a critical part of our analysis. Formally, there is a withholding penalty $\theta > 0$ borne by a manager strategically withholding. Note that it could be that the occurrence of the penalty is random and, under this interpretation, we should define θ as the expected penalty. For obvious practical reasons, we assume that the ability to punish the manager is bounded ($\theta < \bar{v}$); for example, if the manager may have consumed or transferred the misappropriated assets. We also assume that $\phi(\mathbb{E}(V)) - \theta > \phi(0)$ because otherwise there would be no gain from the lowest asset value to withhold.

We need to make assumptions about the problem solved by the measurement. Keeping with the definition used for the baseline model, we examine measurements that maximize the expected price $\mathbb{E}(\phi(\mathbb{E}(V|R)))$, taking aside the personal loss borne by the manager. It is plausible that a regulator

such as the Securities and Exchange Commission (SEC) would care more about productive efficiency than about minimizing the discomfort of managers caught misreporting or we may assume that they focus on investors. But the reason for this criterion is conceptual; we have seen earlier that a fully-informative measurement would be optimal without penalties. If we were to maximize the expected firm value net of expected penalties for strategic withholding, then it would follow by construction that some imprecision is optimal to reduce the probability of bearing penalties. This channel, while it is true and reinforces our current analysis, could be construed as a second-order effect on the objective of the regulator. So in summary, we solve for the optimal imprecision on expected firm value to investors.¹⁷

We begin by observing that withholding penalties need not imply more imprecision because they reduce strategic withholding even in a fully-informative measurement; in fact, as θ becomes large, strategic withholding converges to zero and there remains no scope for imprecision.¹⁸

Lemma 4 *With a fully-precise measurement \mathcal{M}^{all} , there exists a unique solution t_θ given by the implicit solution to*

$$\phi(\mu_{nd}^t) - \theta = \phi(t_\theta). \quad (5)$$

We consider next the design of a measurement which may be different from \mathcal{M}^{all} . The next Lemma is entirely along the lines of Lemmas 1 and 2 so we give it without proof.

¹⁷Another possible assumption would be to assume that the penalty is a deadweight loss taken from firm value, and thus, would reduce the baseline non-disclosure price μ_{nd}^t by the expected penalty. This alternate assumption implies forces very similar to the problem of maximizing manager surplus, except that the penalty is now borne even when information is not received (since the market does not observe strategic withholding). In this setting, it is easily shown that some imprecision is always desirable near the disclosure threshold in order to reduce the probability of the penalty.

¹⁸While entirely intuitive, it is worth noting that there is a difference with the recent result [Dye \(2018\)](#) which develops the opposite intuition that verification over the withholding region would increase withholding. The reason for this difference is that in this earlier study, a key force is that the penalty is paid back by the manager to shareholders, and thus serves as manager-created insurance to investors buying shares in withholding firms. This is an interesting force but in most cases the amount of funds paid directly from managers' pockets is small relative to investor losses, and a large part of the penalty is in the form of losses that are partly deadweight to the parties involved, such as reputations, fines, time spent in defense, lawyer fees or legal penalties. To this point, [Laux and Stocken \(2012\)](#) provide a discussion of how having a deadweight component is essential to providing the right ex-ante incentives. That said, an important joint insight within these papers and our research is that, for the additional forces here to hold, it should be the case that the penalty is not redistributed to shareholders in a way that would affect price in a way that is quantitatively non-trivial.

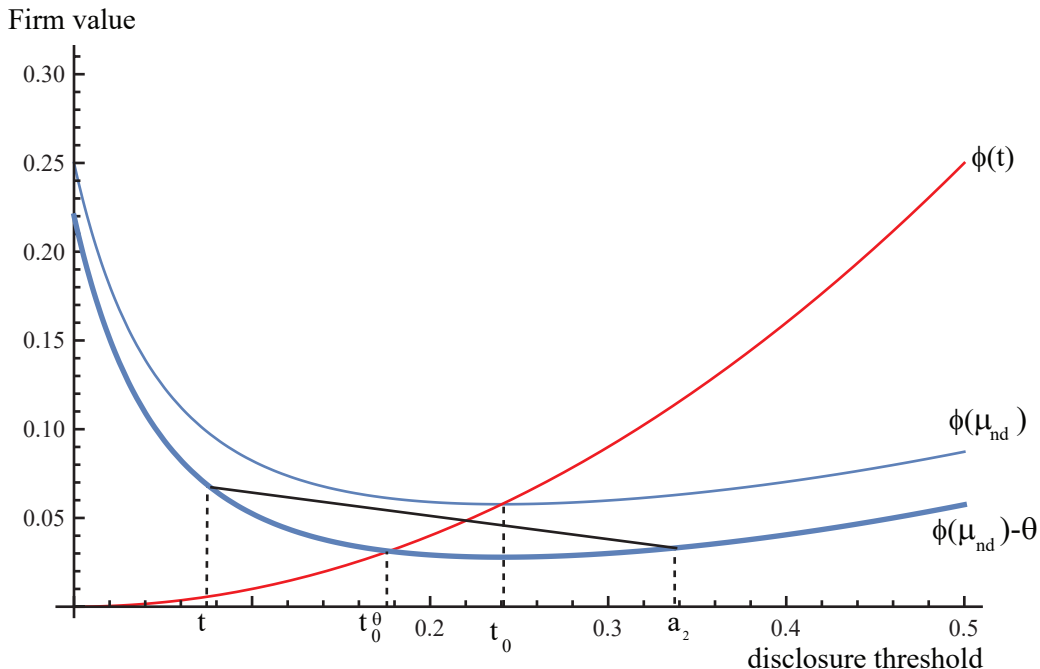


Figure 6: Imprecision with withholding penalties

Lemma 5 Suppose that \mathcal{M} is optimal, then either $\mathcal{M} = \mathcal{M}^{all}$ or $\mathcal{V}_{FI} = (a_2, \bar{v})$. Furthermore,

$$\phi(\mu_{nd}^t) - \theta = \phi(\mathbb{E}(V|V \in [t, a_2))). \quad (6)$$

We next extend Figure 2 to develop more graphical intuition as to how the withholding penalty affects the choice of measurement. We first plot in Figure 6 the payoff to a withholding manager as a function of the threshold t , with and without withholding costs. The solution to Jung and Kwon (1988) with withholding costs is now located at $t_\theta < t_0$. We then apply the previous logic to implement a withholding threshold $t < t_\theta$, with a region of imprecision $[t, a_2)$.¹⁹

Note that, just like for the case of disclosure costs, imprecision creates dispersion in posterior expectations (a pre-condition for the information to be useful in our model) because the non-

¹⁹This is now slightly different from the baseline and the complete solution can no longer be obtained graphically (although the intuition can). As in the baseline, we should try to set a_2 as small possible but large enough so that marginal disclosers do not deviate to withhold, that is, $\phi(\mu_{nd}^t) - \theta = \phi(\mathbb{E}(V|V \in [t, a_2)))$. Using the same construction as in Figure 2, we can recover a threshold $t' > t$ that yields the same posterior μ_{nd}^t and implies (from the same arguments as in Figure 2) that $\mathbb{E}(V|V \in [t, a_2)) = \mu_{nd}^t$. But setting $a_2 = t'$ makes the imprecision excessive since it would imply that $\phi(\mu_{nd}^t) - \theta < \phi(\mathbb{E}(V|V \in [t, a_2)))$. So, as shown in the Figure, the optimal a_2 can be visualized by drawing a line sloping down.

disclosure posterior expectation μ_{nd}^t is strictly greater than the posterior expectation $\mathbb{E}(V|V \in [t, a_2])$. From this, it follows that there would be a strict informational loss when moving t to a_2 into a single withholding region. Indeed, building on this intuition, we derive the following result.

Proposition 3 *With non-zero withholding penalties $\theta > 0$, the optimal measurement exhibits an imprecise region for the marginal discloser $[t^*, a_2^*)$ with $t^* < t_\theta < a_2^*$.*

We can rephrase the intuition for the greater variation in posterior expectations in economic terms. The withholding penalty is now used as a complement to imprecision to discipline firms with asset values higher than t^* to disclose. Under full-information, the penalty would only bind for the marginal discloser which, at $x = t_\theta$, is an event with probability zero. A larger region of imprecision $[t^*, a_2^*)$ implies that the penalty binds for a larger set of asset values.

We discuss below a few additional key implications of the proposition. First, we show that, contrary to standard voluntary disclosure theory, the information becomes endogenously coarse over the lowest reported events. So, we expect in this theory for the firm to be vaguer about bad news that is voluntarily reported. On this point, [Gigler, Kanodia, Sapra and Venugopalan \(2009\)](#) interpret measurement systems that are more precise over favorable information as more conservative. Second, the posterior expectation is not increasing in asset values. In the model, informed managers with sufficiently bad news choose to bear the individual penalty and withhold, but in exchange receive a higher price: formally, the voluntary disclosure problem requires $\phi(\mu_{nd}^{t^*}) - \theta = \phi(\mathbb{E}(V|V \in [t^*, a_2^*]))$ in the manager's problem which, in turn, implies from an investor's perspective that $\phi(\mu_{nd}^{t^*}) > \phi(\mathbb{E}(V|V \in [t^*, a_2^*]))$. A firm choosing to reveal a low asset value would trigger a current market response that is more negative than if it had stayed silent.

We plot in [Figure 7](#) several examples following the uniform specification and quadratic payoff $\phi(\cdot)$ used in the previous section, a probability of being uninformed $q = 0.1$ and various choices for θ from 0 to 0.07. At $\theta = 0$, the expected firm value is maximized at $a_2 = t_0$ which means that there is no value in imprecision. As θ increases, the total value no longer peaks at t_0 and there is an interior imprecise interval on $[a_2^*, t_\theta)$ where a_2^* is the choice that attains the peak of the curve.

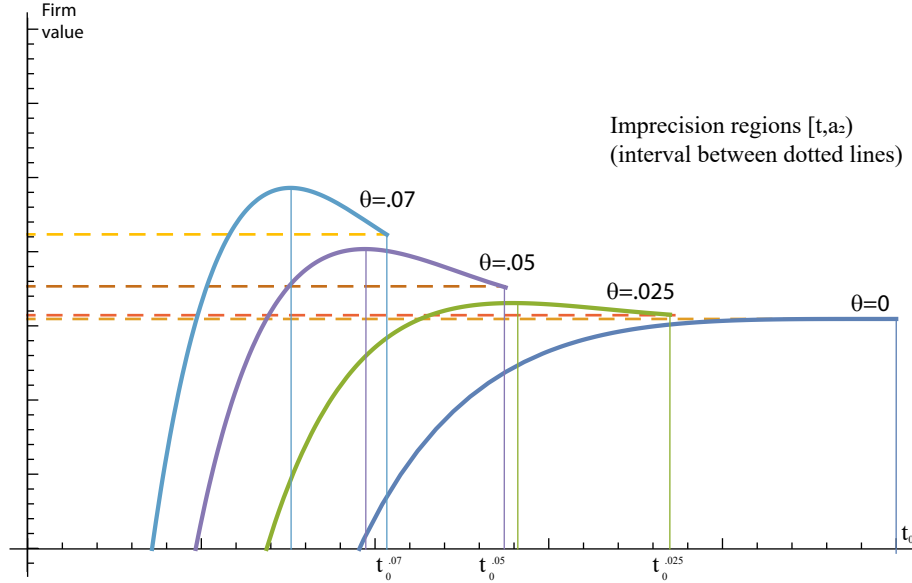


Figure 7: Firm value as a function of imprecision ($q = .1$)

Note that the region of imprecision shifts to the left and shrinks as θ becomes large because then withholding becomes increasingly unlikely. In fact, for values of θ greater than .25, there is no longer any strategic withholding on the equilibrium path and the only firms that do not disclose are those that did not receive information.

4.2 Collateral Financing and Non-convexities

For our baseline analyses, we assumed so far that the firm would, a-priori, prefer to implement measurements with more information after the disclosure choice. Here, we revisit an application of the theory to the financing problem described in [Goex and Wagenhofer \(2009\)](#) and [Bertomeu and Cheyrel \(2015\)](#), in which the firm uses the measured asset as collateral to raise external funds. Goex and Wagenhofer show that, absent voluntary disclosure, the measurement would prescribe imprecision over favorable events to maximize the likelihood of meeting a collateral constraint.²⁰ We extend this problem along the two dimensions specific to our study: first incorporating voluntary disclosure with a friction to receiving information, and second, considering the potential

²⁰We also derived (on request) a theoretical solution for an abstract class of S-shaped payoff function $\phi''' < 0$ which imply a very similar measurement system as in this section.

value of information after the firm receives financing affected by both measurement and voluntary disclosures.²¹

As in [Goex and Wagenhofer \(2009\)](#), the firm must finance an investment to operate but in order to do so, pledges a minimum value of assets as collateral in the form of a constraint $\mathbb{E}(V|r) \geq \hat{v}$, where \hat{v} is a collateral requirement; see [Holmström and Tirole \(1997\)](#) or [Goex and Wagenhofer \(2009\)](#) for various micro-foundations in terms of a principal-agent problem.²² If the firm does not operate, it obtains a payoff normalized to zero, implying a payoff $\phi(x) = 1_{x \geq \hat{v}}\psi(x)$ where $\psi(x) \geq \psi(0) = 0$ is a convex payoff once the investment is made.

Note that, if $\hat{v} > 0$, the payoff is neither convex nor concave and, as $\hat{v} \rightarrow 0$, the model reduces to the baseline problem. For later use in this section, it is convenient to denote t_0 as the voluntary disclosure threshold defined earlier absent any collateral constraint, that is with $\phi(x) = \psi(x)$.

We shall derive a solution by decomposing the design problem in terms of two possible choices of measurement. The first family of measurements is one in which some firms do not meet their collateral constraint. This can only occur if disclosure is a pre-condition to receiving financing, that is, $\mu_{nd}^t < \hat{v}$, since otherwise no informed firm would willingly disclose information that would cause its value to fall below the collateral requirement. In the next lemma, we solve for the preferred measurement in this family.

Lemma 6 *Let \mathcal{M}^1 be an optimal measurement in which some firms do not meet their collateral constraint. Then it takes the form of a withholding threshold t and a collateral threshold $a_2 > t$, such that:*

- (i) *asset values below t are withheld with $\mu_{nd}^t < \hat{v}$;*

²¹This formulation is a slightly modified version of [Goex and Wagenhofer \(2009\)](#) - part of this is to nest our baseline model with convex payoffs in this analysis but there is also a fundamental reason as to why additional generality is important here. In their original model, Goex and Wagenhofer assume a fixed project payoff $\psi(x) = \pi_0$ when it is financed so that the optimal decision that is implemented by the measurement is binary. The binary decision makes the voluntary disclosure problem moot because financed firms would always disclose voluntarily. [Bertomeu and Cheynel \(2015\)](#) develop this model when the proceeds from selling the collateral can be reinvested, so that $\phi(\cdot)$ is linear in parts and do not model the interaction between measurement and voluntary disclosure.

²²These studies show that \hat{v} can be recovered as a function, for example, of the cost of effort of the agent and the properties of cash flows and accounting reports.

(ii) asset values are reported in the form of an imprecise measurement $[t, a_2)$ that exactly meets the collateral constraint $\mathbb{E}(V|V \in [t, a_2)) = \hat{v}$; and

(iii) asset values $v \geq a_2$ are perfectly measured and disclosed.

A point of note is that the optimal choice of $[t, a_2)$ is a function of the benefit of meeting the collateral constraint $\psi(\hat{v})$ and the convexity of the function $\psi(\cdot)$. To be specific, the choice of the imprecise region $[t, a_2)$ maximizes the expected value of the firm

$$\begin{aligned} \sigma = & (1 - q)(F(a_2) - F(t))\psi(\mathbb{E}(V|V \in [t, a_2))) \\ & + (1 - q) \int_{a_2}^{\bar{v}} f(x)\psi(x)dx, \end{aligned} \tag{7}$$

subject to the collateral constraint $\mathbb{E}(V|V \in [t, a_2)) = \hat{v}$.

If $\psi(\cdot)$ is approximately linear to the right \hat{v} , we return to the type of measurements in the spirit of [Goex and Wagenhofer \(2009\)](#) in which $a_2 = \bar{v}$ is the preferred measurement to use when not all firms can meet their collateral constraints. At the other extreme, if \hat{v} is small, we return to the convex problem in proposition 1, and the optimal measurement features a_2 nearly equal to t . Even in this case, however, the optimal choice a_2 is never exactly equal to t , which rules out fully informative measurements of the form derived earlier.

We next consider a second family of measurements in which all firms meet their collateral constraints within the chosen measurement. Note that, for this to occur, withholding firms must achieve a posterior expectation $\mu_{nd}^t \geq \hat{v}$. We know that $\mu_{nd}^t \in (\mu_{nd}^{t_0}, \mathbb{E}(V)]$, so this equation can be met for some t if and only if the unconditional expected value of the asset $\mathbb{E}(V)$ is greater than \hat{v} . We solve for the optimal measurement within this family in the next lemma.

Lemma 7 *Let \mathcal{M}^2 be an optimal measurement in which all firms are financed:*

(i) *if $\hat{v} \leq t_0$, the full-information measurement \mathcal{M}^{FI} is optimal;*

(ii) *if $\hat{v} \in (\mu_{nd}^{t_0}, \mathbb{E}(V)]$, there exists $t > t_0$ given by $\mu_{nd}^t = \hat{v}$ such that \mathcal{M}^2 has two regions: all asset values below t are withheld, and all asset values above t are disclosed;*

(iii) If $\hat{v} \geq \mathbb{E}(V)$, there is no measurement in which all firms are financed.

The setting in (i) is a special case of the model where the unconditional value of the collateral is sufficiently large so that strategic withholding would on its own imply a non-disclosure posterior expectation that would meet the collateral constraint. Note that the higher the probability that the measurement does not yield information, the greater the non-disclosure price and therefore the lower the collateral constraint can be. When this case occurs, the imprecision created by the voluntary disclosure problem entirely subsumes any additional amount of imprecision in the measurement. Hence, the firm can simply use a full-information measurement and let the manager strategically withhold information on an ex-post basis.

Part (ii) extends this argument to the richer setting in which a full-information measurement would have caused non-disclosing firms to shut down. To address the financing problem, the measurement pools uninformed firms and informed firms with $v < t$ until the collateral requirement $\mu_{nd}^t = \hat{v}$ is met. This requires, in turn, to set $t > t_0$ and raises imprecision above the level that would have been implemented absent collateral requirements.

Extending the comparative static in (i), an increase in the probability to receive information will reduce μ_{nd}^t as investors become skeptical. To compensate for this decrease, the reporting threshold t must increase. Put differently, precision in the measurement and in voluntary disclosure act as substitutes, with greater imprecision being implemented in response to a lower friction in the voluntary disclosure channel. Finally, in part (iii), if the unexpected value of the collateral $\mathbb{E}(V)$ falls below \hat{v} , not all firms can be financed and a measurement in this family cannot be used.

When is it desirable to use each of the two types measurement? The next proposition develops a direct comparison of the two families of measurements as a function of the severity of the collateral problem and the probability of information endowment.

Proposition 4 *The optimal measurement is as follows:*

(i) If $\hat{v} < \mu_{nd}^{t_0}$, a fully-informative measurement \mathcal{M}^{all} is an optimal measurement.

(ii) *Otherwise, there exists $q_0 \in [0, 1]$ such that if $q < q_0$, the optimal measurement has the form \mathcal{M}^1 in which non-disclosing firm cannot be financed and, if $q \geq q_0$, the optimal measurement has the form \mathcal{M}^2 in which all firms are financed.*

A few comments are helpful to interpret the proposition in practical terms. If the collateral constraint is sufficiently severe, the optimal measurement must focus on increasing the collateral above the minimum required collateral \hat{v} . In turn, this requires taking away financing from certain firms with $v < \hat{v}$. But the voluntary disclosure problem implies that such firms could have claimed to be uninformed, so that this type of solution to the financing problem involves denying financing to all uninformed firms, which can be particularly costly when assets are hard to measure and it is likely that the firm does not receive information.

If the collateral constraint is moderate or mild, it is possible to finance all firms by creating enough imprecision in the measurement but doing so comes with a trade-off because it removes useful information when firms are financed. There are, therefore, two possible options to design the measurement. One is to finance all firms and tolerate a relatively coarse measurements that includes uninformed firms and firms with $v \leq \hat{v}$. The other is to use a measurement in which non-disclosure firms do not receive financing but, conditional on being financed, firms report a precise measurement.

The main consideration required to select between the two types of measurements is the probability of receiving information. To explain this, note first that for all firms to be financed in \mathcal{M}^2 , it must be that $\mathbb{E}(v) \geq \hat{v}$ so that there is enough asset value in expectation. In turn, this implies that, in any setting in which we may elect \mathcal{M}^2 , it will be the case that firms that do not receive verifiable information, whose expected asset value is $\mathbb{E}(V)$ contribute to meeting the collateral constraint. The higher q , the higher the proportion of such firms in forming the non-disclosure expectation μ_{nd}^t and thus the easier it will be to meet the collateral constraint in \mathcal{M}^2 . By contrast, the higher q , the greater the fraction of non-disclosing firms that cannot invest in a measurement \mathcal{M}^1 . So, considering both forces, environments with less information are conducive to measurements which favor coarse measurements for low asset values and more financing.

5 Conclusion

By and large, prior literature emphasizes either accounting measurements as ex-ante choices or the problem of strategic reporting over an otherwise external arrival of information. Yet, both problems are closely tied to each other, with the ex-ante design affecting incentives to withhold information. What can we say of the nature of measurements and voluntary disclosures when both are endogenously determined?

In our baseline model, there is uncertainty about whether objective information exists about a particular outcome and the firm designs a measurement system that may publicly provide information when objective information exists. As a broad example, an event may or may not have occurred or may be measurable, and the firm may design a measurement system that collects information about the event provided measurable information exists. In this environment, we find that the firm should perfectly measure its assets. There is a loss of information due to strategic withholding at the disclosure stage and suitably-designed measurements could certainly reduce the probability of withholding. But the price to pay to do so is too large in that the required imprecision required would destroy any possible use of this information.

We have shown this insight within a fairly generic version of the [Dye \(1985\)](#) and [Jung and Kwon \(1988\)](#) and an intuitive class of decision problems in which, after the report is observed, management makes the decision that maximizes post-report market prices. Yet we view it as a first benchmark to uncovering elements that would lead to imprecision becoming valuable in this setting. We study two empirically plausible extensions of the baseline model. First, as a function of private penalties imposed on management, some imprecision around the voluntary disclosure threshold always increases firm value. Second, if the firm bears collateral constraints, there is value in imprecision as a function of disclosure frictions. We can take these results as broad implications as to the nature of designing accounting rules, with attention to the voluntary nature of the process through which information flows to the market.

Applied questions may provide paths for continuing the research agenda discussed in this study, with the focus on non-convexities due to collateral measurements being one of many potential av-

enues. How do measurements and voluntary disclosures interact when the firm wishes to persuade an auditor to issue a favorable opinion? How does the firm select measurements to control information flows to the product market? We hope that these classic questions can be revisited from the perspective of a theory in which measurement and voluntary disclosures endogenously coexist.

Appendix

A Proofs

Proof of Lemma 1. Let \mathcal{M} be a measurement system. Recall that we restrict the attention to \mathcal{V}_{FI} being an interval and, without loss of generality, set the withholding region $[a_0, t]$ not to overlap with \mathcal{V}_{FI} . We already know that asset values in $[a_0, t]$ are withheld, so that \mathcal{V}_{FI} must be either empty, must be the interval $[t, a_2)$ or must be some interval $[a_i, a_{i+1})$ with $i \geq 2$.

Suppose by contradiction that \mathcal{M} does not have the form prescribed in Lemma 1. We construct an alternative measurement system $\mathcal{M}' = (\mathcal{V}'_{FI}, (a'_i)_{i=0}^I)$, with associated report R' . Let us set \mathcal{M}' to coincide with \mathcal{M} except for $\mathcal{V}'_{FI} = \mathcal{V}_{FI} \cup [a_2, \bar{v})$, which implies (i) the requirement of (2) remains satisfied by \mathcal{M}' and R' is more precise in the sense of Blackwell than r . By Jensen's inequality, this implies that

$$\mathbb{E}(\phi(\mathbb{E}(V|R))) < \mathbb{E}(\phi(\mathbb{E}(V|R'))),$$

and violates the optimality of \mathcal{M} . It follows that any \mathcal{M} with $I \geq 3$ must be such that $\mathcal{V}_{FI} = [a_2, \bar{v})$. ■

Proof of Lemma 2. Let \mathcal{M} be an optimal measurement. We know that if the claim does not hold, we know from lemma 1 that $[t, a_2) \neq \mathcal{V}_{FI}$ is a coarse interval and, from (2), is such that $\mu_{nd}^t \leq \mathbb{E}(V|V \in [t, a_2))$.

Suppose by contradiction that $\mu_{nd}^t > \mathbb{E}(V|V \in [t, a_2))$. By continuity, there exists $y < a_2$ such that $\mu_{nd}^t > \mathbb{E}(V|V \in [t, y])$. Consider a measurement \mathcal{M}' which coincides with \mathcal{M} except that the element of the partition $[t, a_2)$ is replaced by two elements $[t, y)$ and $[y, a_2)$. Note that the withholding region is unchanged, implying that μ_{nd}^t remains unchanged and, by construction of y , (2) is satisfied. However,

\mathcal{M}' implies more precise information in the sense of Blackwell and, therefore, from Jensen's inequality, $\mathbb{E}(\phi(\mathbb{E}(V|R')))) > \mathbb{E}(\phi(\mathbb{E}(V|R)))$, contradicting the optimality of \mathcal{M} . ■

Proof of Lemma 3. Let \mathcal{M} be an optimal measurement. By contradiction, if the claim does not hold, we know from lemma 1 that $[t, a_2) \neq \mathcal{V}_{FI}$ is a coarse interval and, from (2), is such that $\mu_{nd}^t \leq \mathbb{E}(V|V \in [t, a_2))$.

Suppose by contradiction that $\mu_{nd}^t > \mathbb{E}(V|V \in [t, a_2))$. By continuity, there exists $y < a_2$ such that $\mu_{nd}^t > \mathbb{E}(V|V \in [t, y))$. Consider a measurement \mathcal{M}' which coincides with \mathcal{M} except that the element of the partition $[t, a_2)$ is replaced by two elements $[t, y)$ and $[y, a_2)$. Note that the withholding region is unchanged, implying that μ_{nd}^t remains unchanged and, by construction of y , (2) is satisfied. However, \mathcal{M}' implies more precise information in the sense of Blackwell and, therefore, from Jensen's inequality, $\mathbb{E}(\phi(\mathbb{E}(V|R')))) > \mathbb{E}(\phi(\mathbb{E}(V|R)))$, contradicting the optimality of \mathcal{M} .

It thus follows from the previous paragraph that $\mu_{nd}^t = \mathbb{E}(V|V \in [t, a_2))$. But, then, we can create a measurement \mathcal{M}' by setting $t' = a_1' = a_2$, leaving all the other elements of the partition in \mathcal{M} unchanged.

We then know that

$$\begin{aligned} \mu_{nd}^{t'} &= \frac{q\mathbb{E}(V) + (1-q)F(a_2)\mathbb{E}(V|V \leq a_2)}{q + (1-q)F(a_2)} \\ &= \frac{q + (1-q)F(t)}{q + (1-q)F(a_2)} \times \frac{q\mathbb{E}(V) + (1-q)F(t)\mathbb{E}(V|V \leq t)}{q + (1-q)F(t)} \\ &\quad + \frac{(1-q)(F(a_2) - F(t))}{q + (1-q)F(a_2)} \times \mathbb{E}(V|V \in [t, a_2)) \\ &= \frac{q + (1-q)F(t)}{q + (1-q)F(a_2)} \mu_{nd}^t + \frac{(1-q)(F(a_2) - F(t))}{q + (1-q)F(a_2)} \mu_{nd}^t = \mu_{nd}^t. \end{aligned}$$

To satisfy (2) in \mathcal{M}' , we are left to compare $\mu_{nd}^{t'}$ and a_2 ; however, since $\mu_{nd}^t = \mathbb{E}(V|V \in [t, a_2))$, we know that $a_2 \geq \mu_{nd}^t$. To conclude the proof, note that, by construction, $\mathbb{E}(V|R)$ and $\mathbb{E}(V|R')$ are always equal, hence, \mathcal{M}' yields the same payoffs as \mathcal{M} . ■

Proof of Proposition 1. We have shown in lemma 3 that we can restrict the analysis to measurements where $\mathcal{V}_{FI} = [t, \bar{v})$. From (2), it must hold that

$$\mu_{nd}^t = \frac{q\mathbb{E}(V) + (1-q)F(t)\mathbb{E}(V|V \leq t)}{q + (1-q)F(t)} \leq t.$$

Integrating by parts as in [Jung and Kwon \(1988\)](#) and rearranging terms, this inequality is rewritten as

$$\psi(t) = (1 - p) \int_0^t F(v) dv - p(\mathbb{E}(V) - t) \geq 0. \quad (\text{A.1})$$

Recall from earlier that this takes the form of an inequality because while a firm with $v \geq t$ could withhold, the firm with $v < t$ receives a message $[0, t)$ from the measurement and would never do better than withhold. Differentiating this expression,

$$\psi'(t) = (1 - p)F(t) + p > 0,$$

and we also know from [Jung and Kwon \(1988\)](#) that $\psi(t_0) = 0$. So, (A.1) is satisfied if and only if $t \geq t_0$.

Having shown this, comparing a threshold $t > t_0$ to t_0 , we know that the latter is more precise in the sense of Blackwell, hence, it yields higher expected surplus. To conclude the argument, note that a fully informative measurement \mathcal{M}^{all} implies that equation (2) must be satisfied at equality and implements t_0 . This concludes the proof and establishes that \mathcal{M}^{all} , that is, full-information is optimal. ■

Proof of Proposition 2. For a given threshold t , the upper bound of the imprecise interval, a_2 , satisfies the indifference condition for the marginal discloser, that is,

$$\Gamma(t, a_2) \equiv \phi(\mathbb{E}(V|V \in [t, a_2])) - c - \phi(\mu_{nd}^t) = 0. \quad (\text{A.2})$$

The expected firm value, denoted Σ , is given by

$$\begin{aligned} \Sigma = & (q + (1 - q)F(t))\phi(\mu_{nd}^t) + (1 - q)(F(a_2) - F(t))\phi(\mathbb{E}(V|V \in [t, a_2])) \\ & + (1 - q) \int_{a_2}^{\bar{v}} \phi(v) f(v) dv. \end{aligned} \quad (\text{A.3})$$

Taking the derivative of the expected firm value in (A.3) with respect to t yields

$$\begin{aligned} \frac{\partial \Sigma}{\partial t} = & (1 - q)f(t)\phi(\mu_{nd}^t) + (q + (1 - q)F(t))\frac{\partial \mu_{nd}^t}{\partial t}\phi'(\mu_{nd}^t) \\ & + (1 - q)(F(a_2) - F(t))\phi'(\mathbb{E}(V|V \in [t, a_2]))\frac{\partial \mathbb{E}(V|V \in [t, a_2])}{\partial t} \\ & + (1 - q)\left(\frac{\partial a_2}{\partial t}f(a_2) - f(t)\right)\phi(\mathbb{E}(V|V \in [t, a_2])) - (1 - q)\frac{\partial a_2}{\partial t}\phi(a_2)f(a_2). \end{aligned} \quad (\text{A.4})$$

We obtain separately the following derivatives in (A.4),

$$\begin{aligned}
\frac{\partial \mathbb{E}(V|V \in [t, a_2])}{\partial t} &= \left(\frac{\partial a_2}{\partial t} a_2 f(a_2) - t f(t) \right) / (F(a_2) - F(t)) \\
&\quad - \left(\frac{\partial a_2}{\partial t} f(a_2) - f(t) \right) \int_t^{a_2} x f(x) dx / \left(F(a_2) - F(t) \right)^2 \\
&= \left(\frac{\partial a_2}{\partial t} f(a_2) (a_2 - \mathbb{E}(V|V \in [t, a_2])) \right) / (F(a_2) - F(t)) \\
&\quad - f(t) (t - \mathbb{E}(V|V \in [t, a_2])) / (F(a_2) - F(t)), \\
\frac{\partial \mu_{nd}^t}{\partial t} &= \frac{(1-q)f(t)(t - \mu_{nd}^t)}{q + (1-q)F(t)}.
\end{aligned}$$

Reinjecting the expressions above into (A.4) yields

$$\begin{aligned}
\frac{\partial \Sigma}{\partial t} &= (1-q)f(t) \left(\phi'(\mu_{nd}^t)(t - \mu_{nd}^t) - (\phi(t) - \phi(\mu_{nd}^t)) \right) \\
&\quad + (1-q)f(a_2) \frac{\partial a_2}{\partial t} \left(\phi'(\mathbb{E}(V|V \in [t, a_2]))(a_2 - \mathbb{E}(V|V \in [t, a_2])) - \left(\phi(a_2) - \phi(\mathbb{E}(V|V \in [t, a_2])) \right) \right) \\
&\quad + (1-q)f(t) \left(\phi'(\mathbb{E}(V|V \in [t, a_2]))(\mathbb{E}(V|V \in [t, a_2]) - t) - (\phi(\mathbb{E}(V|V \in [t, a_2])) - \phi(t)) \right), \quad (\text{A.5})
\end{aligned}$$

where we have subtracted $(1-q)f(t)\phi(t)$ in the first term and added it back in the third.

Using the implicit function theorem on $\Gamma(t, a_2) = 0$ from (A.2), we find that the expression for $\frac{\partial a_2}{\partial t}$ is

$$\begin{aligned}
\frac{\partial a_2}{\partial t} &= - \frac{\partial \Gamma(t, a_2)}{\partial t} / \frac{\partial \Gamma(t, a_2)}{\partial a_2} \\
&= \frac{\frac{\phi'(\mu_{nd}^t)}{\phi'(\mathbb{E}(V|V \in [t, a_2]))} \frac{\partial \mu_{nd}^t}{\partial t} [F(a_2) - F(t)] - f(t) [\mathbb{E}(V|V \in [t, a_2]) - t]}{f(a_2) [a_2 - \mathbb{E}(V|V \in [t, a_2])]} . \quad (\text{A.6})
\end{aligned}$$

We first show that (A.5) evaluated at $t = t_c$ is negative. As $t \rightarrow t_c$, the imprecise interval shrinks, that is, $a_2 \rightarrow t$ because the lower and upper bounds of the imprecise interval both converge to t_c . Therefore, in this limit the third term of (A.5) converges to zero. Also the second term converges to zero, since $\frac{\partial a_2}{\partial t} \Big|_{t=t_c}$

is finite.²³ Only the first term is nonzero in the limit, hence

$$\frac{\partial \Sigma}{\partial t} \Big|_{t=t_c} = (1-q)f(t_c) \left(\phi'(\mu_{nd}^{t_c})(t_c - \mu_{nd}^{t_c}) - (\phi(t_c) - \phi(\mu_{nd}^{t_c})) \right).$$

The indifference condition under perfect measurement, $\phi(t_c) - c = \phi(\mu_{nd}^{t_c})$, implies that $t_c > \mu_{nd}^{t_c}$, so by convexity of $\phi(\cdot)$ we have $\phi(t_c) > \phi(\mu_{nd}^{t_c}) + \phi'(\mu_{nd}^{t_c})(t_c - \mu_{nd}^{t_c})$. We conclude that

$$\frac{\partial \Sigma}{\partial t} \Big|_{t=t_c} < 0.$$

As a result, setting $t = t_c$ is not optimal and reducing $t < t_c$ will increase the expected firm value.

After we have shown that precise measurement is suboptimal, there remains to argue that an optimal measurement exists. Setting $t > t_c$ is suboptimal, as it both increases strategic withholding and reduces the set of perfectly measured asset values that are disclosed. Inspection of (A.5) shows that $\frac{\partial \Sigma}{\partial t} > 0$ for all $t \leq t_0$, so setting $t \leq t_0$ is also suboptimal. Taken together, these considerations imply that there exists a maximizer t^* in the interval (t_0, t_c) . ■

Proof of Lemma 4. The non-disclosure price μ_{nd}^t remains given by equation (2) and, after applying the integration by parts from Jung and Kwon (1988), can be written as

$$\mu_{nd}^t = \frac{q(\mathbb{E}(V) - tF(t)) + tF(t) - \int_0^t F(v)dv}{q + (1-q)F(t)}.$$

²³The expression for $\frac{\partial a_2}{\partial t} \Big|_{t=t_c}$ can be derived as follows. First, note that

$$\frac{F(a_2) - F(t)}{f(a_2)[a_2 - \mathbb{E}(V|t \leq V \leq a_2)]} = \frac{[F(a_2) - F(t)]^2}{f(a_2) \{a_2[F(a_2) - F(t)] - \int_t^{a_2} xf(x)dx\}}$$

and

$$\frac{f(t)[\mathbb{E}(V|t \leq V \leq a_2) - t]}{f(a_2)[a_2 - \mathbb{E}(V|t \leq V \leq a_2)]} = \frac{f(t) \{ \int_t^{a_2} xf(x)dx - t[F(a_2) - F(t)] \}}{f(a_2) \{ a_2[F(a_2) - F(t)] - \int_t^{a_2} xf(x)dx \}}.$$

Second, by L'Hôpital's rule,

$$\lim_{a_2 \rightarrow t} \frac{[F(a_2) - F(t)]^2}{f(a_2) \{ a_2[F(a_2) - F(t)] - \int_t^{a_2} xf(x)dx \}} = \lim_{a_2 \rightarrow t} \frac{1}{f(a_2)} \lim_{a_2 \rightarrow t} \frac{2[F(a_2) - F(t)]f(a_2)}{F(a_2) - F(t)} = 2$$

and

$$\lim_{a_2 \rightarrow t} \frac{f(t) \{ \int_t^{a_2} xf(x)dx - t[F(a_2) - F(t)] \}}{f(a_2) \{ a_2[F(a_2) - F(t)] - \int_t^{a_2} xf(x)dx \}} = \lim_{a_2 \rightarrow t} \frac{f(t)}{f(a_2)} \lim_{a_2 \rightarrow t} \frac{f(a_2)(a_2 - t)}{F(a_2) - F(t)} = \lim_{a_2 \rightarrow t} f(a_2) \lim_{a_2 \rightarrow t} \frac{a_2 - t}{F(a_2) - F(t)} = 1.$$

The equilibrium must then be a solution to the indifference condition

$$\phi(\mu_{nd}^t) - \theta = \phi(t),$$

where the left-hand side is the payoff from withholding net of penalty and the right-hand side is the payoff to the marginal discloser. We show next the existence and uniqueness of the solution.

To show existence, evaluate first at $t = t_0$ which implies that $\mu_{nd}^t = t_0$ and, therefore, $\phi(\mu_{nd}^t) - \theta < \phi(t)$ when $t = t_0$. Evaluate next at $t = 0$, in which case $\mu_{nd}^t = \mathbb{E}(V)$, which implies (by assumption) that $\phi(t) - \theta > \phi(t)$ when $t = 0$. So, at least one solution exists with $t_\theta \in (0, t_0)$.

To show uniqueness, we first need to demonstrate that μ_{nd}^t is concave in t , with minimum at t_0 . This is a small elaboration on the claim in Proposition 1 of [Acharya, DeMarzo and Kremer \(2011\)](#) who show only the second part of this claim. To show this, rewrite (2) after applying the standard integration by parts, to obtain

$$\mu_{nd}^t = \frac{q(\mathbb{E}(V) - tF(t)) + tF(t) - \int_0^t F(v)dv}{q + (1 - q)F(t)},$$

and differentiate in t ,

$$\frac{\partial \mu_{nd}^t}{\partial t} = \frac{(1 - q)f(t)[(1 - q) \int_0^t F(v)dv - q(\mathbb{E}(V) - t)]}{(q + (1 - q)F(t))^2}.$$

The term in brackets defines the threshold t_0 when set to zero and changes sign as t crosses t_0 . In turn, this implies that μ_{nd}^t is U-shaped in t , as claimed.

We conclude that $\phi(\mu_{nd}^t)$ decreases for t on $[0, t_0]$, implying at most one solution for t_θ on the interval $(0, t_0)$. We also know that $\mu_{nd}^t > t$ for $t \in (t_0, \bar{v})$ from the uniqueness of a solution to the standard [Jung and Kwon \(1988\)](#) model. Hence, the solution is unique and must be located somewhere in $(0, t_0)$. ■

Proof of Proposition 3. The claim follows from the proof of Proposition 2 by symmetric arguments. ■

Proof of Lemma 6. We first show by contradiction that there must be an imprecise region $[t, a_2)$ such that $\mathbb{E}(V|V \in [t, a_2)) \geq \hat{v}$. There are two cases to dismiss.

Case 1. There is an imprecise region $[t, a_2)$ but it is such that $\mathbb{E}(V|V \in [t, a_2)) < \hat{v}$ so that firms in this region do not meet the collateral constraint. But, then, we can define a payoff equivalent measurement with

a voluntary disclosure threshold set at $t = a_2$. We can repeat this argument if $[a_2, a_3)$ has the same property.

Case 2. Note that $t > \hat{v}$ cannot be optimal since it yields lower expected firm value than setting $t = \hat{v}$ (all firms achieve the same payoff, except those with $x \in [\hat{v}, t)$ which now meet the collateral constraint). We are left to improve over a measurement such that $t = \hat{v}$ and $\mathcal{V}_{FI} = [\hat{v}, \bar{v}]$.²⁴

We take the derivative on the objective function (7) with respect to t . It yields

$$(1 - q) \left(f(a_2) \frac{\partial a_2}{\partial t} - f(t) \right) \psi(\hat{v}) - (1 - q) f(a_2) \psi(a_2) \frac{\partial a_2}{\partial t}. \quad (\text{A.7})$$

As $t \uparrow \hat{v}$, $a_2 \downarrow \hat{v}$, the derivative simplifies to:

$$- (1 - q) f(\hat{v}) \psi(\hat{v}) < 0 \quad (\text{A.8})$$

which confirms that for t set sufficiently close to \hat{v} , the imprecise region $[t, a_2)$ will increase the expected value of the firm.

We then know from Case 1 and 2 that there exists an imprecise region $[t, a_2)$; further, because $\mathbb{E}(V|V \in [t, a_2)) > \hat{v}$, it must be that $a_2 > \hat{v}$ and therefore $\phi(\cdot)$ is convex for any $v \geq a_2$. A direct application of Jensen's inequality implies that setting $\mathcal{V}_{FI} = [a_2, \bar{v})$ is preferred to any other choice in the measurement.

We are left to show that $\mathbb{E}(V|V \in [t, a_2)) = \hat{v}$. Suppose not, and for expositional purposes, let us denote $M = \mathbb{E}(V|V \in [t, a_2))$. Differentiating the total surplus Σ in (7) with respect to a_2 ,

$$\frac{\partial \Sigma}{\partial a_2} = f(a_2) [\Psi(M) - \Psi(a_2) + (a_2 - M) \Psi'(M)].$$

We know from the convexity of $\phi(\cdot)$ on $y \geq M$ that

$$\Psi'(M) < \frac{\Psi(a_2) - \Psi(M)}{a_2 - M}.$$

which implies that this derivative is negative and the expected value would be increased by decreasing a_2 , a contradiction. ■

²⁴To be rigorous, we do not claim that this measurement would be feasible (it could be that $\mu_{nd}^t > \hat{v}$) but, ignoring this constraint, it is an upper bound on any measurement with the form given in case 2. Hence, by showing that some imprecision does better than this measurement, we know that case 2 cannot be optimal either.

Proof of Lemma 7. For part (i), we can solve a relaxed problem with $\phi(x) = \psi(x)$, ignoring the collateral constraint. This relaxed problem yields an upper bound on the expected firm value achievable in the problem with $\hat{v} > 0$. We know from Proposition 1 that the solution to this problem is the full-information \mathcal{M}^{all} and implies a posterior expectation conditional on withholding given by t_0 . Hence, this measurement would meet the collateral constraint conditional on withholding in the original problem.

To show part (ii), note that we need $\mu_{nd}^t \geq \hat{v}$. We have shown in the proof of lemma 4 that μ_{nd}^t is U-shaped with minimum at t_0 and equals $\mathbb{E}(V)$ at $t = 0$ or, in the limit when $t \rightarrow \bar{v}$. Hence, the condition can be rewritten as either $t \leq b_0$ or $t \geq b_1$, where $b_0 \leq t_0 \leq b_1$. We also know from the proof of lemma 3 that two measurements with threshold t, t' such that $t < t_0 < t'$ and $\mu_{nd}^t = \mu_{nd}^{t'}$ yield the same expected firm value, so we can, without loss of generality, restrict the analysis to $t \geq b_1$. An optimal measurement should then prescribe full-information for any asset values $v \geq t$. If $t > b_1$, that is $\mu_{nd}^t > \hat{v}$ is not binding, reducing t would be feasible and increase the full-information region. In turn, from the convexity of $\phi(\cdot)$ and applying Jensen's inequality, this will increase expected firm value. It follows that $\mu_{nd}^t = \hat{v}$ is binding.

The proof for part (iii) follows from the law of iterated expectation: for all firms to be financed, it must hold that $\mathbb{E}(V|R) \geq \hat{v}$ for any equilibrium report R , which implies $\mathbb{E}(\mathbb{E}(V|R)) = \mathbb{E}(V) \geq \hat{v}$. ■

Proof of Proposition 4. The proof of (i) follows directly from the proof of Lemma 7 (i) given that \mathcal{M}^{all} solves the relaxed problem in which the collateral constraint is ignored.

For (ii), we know from Lemma 7 (iii) that only a measurement of the form \mathcal{M}^1 is feasible when $\hat{v} > \mathbb{E}(V)$, so let us focus on the case $\hat{v} \leq \mathbb{E}(V)$. We denote Σ_1 as the expected firm value with a measurement of the form \mathcal{M}^1 with disclosure threshold t_1 , and Σ_2 as the expected firm value with a measurement of the form \mathcal{M}^2 with disclosure threshold t_2 . Letting $\Delta = \Sigma_1 - \Sigma_2$.

Rewriting:

$$\begin{aligned} \Delta = & (1 - q) \left(F(a_2) - F(t_1) \right) \psi(\hat{v}) + (1 - q) \int_{a_2}^{\bar{v}} \psi(v) f(v) dv \\ & - (q + (1 - q)F(t_2)) \psi(\hat{v}) - (1 - q) \int_{t_2}^{\bar{v}} \psi(v) f(v) dv. \end{aligned}$$

Differentiating with respect to q ,

$$\begin{aligned} \frac{\partial \Delta}{\partial q} &= -\left(F(a_2) - F(t_1)\right)\psi(\hat{v}) - \int_{a_2}^{\bar{v}} \psi(v)f(v)dv - \left(1 - F(t_2)\right)\psi(\hat{v}) \\ &\quad + \int_{t_2}^{\bar{v}} \psi(v)f(v)dv + (1 - q)\frac{\partial t_2}{\partial q}\psi(t_2)f(t_2). \end{aligned} \quad (\text{A.9})$$

Applying the implicit function theorem,

$$\frac{\partial t_2}{\partial q} = -\frac{\partial H(t_2)/\partial q}{\partial H(t_2)/\partial t_2}, \quad (\text{A.10})$$

where $H(t_2) = \mu_{nd}^{t_2} - \hat{v}$. Further,

$$\frac{\partial H(t_2)}{\partial q} = \frac{F(t_2)\mathbb{E}(V) - \int_0^{t_2} vf(v)dv}{\left(q + (1 - q)F(t_2)\right)^2} > 0$$

and

$$\frac{\partial H(t_2)}{\partial t_2} = \frac{(1 - q)f(t_2)(t_2 - \mu_{nd}^{t_2})}{q + (1 - q)F(t_2)} > 0.$$

Hence, $\frac{\partial t_2}{\partial q} < 0$. Moreover, when $\Delta = 0$,

$$\begin{aligned} \int_{a_2}^{\bar{v}} \psi(v)f(v)dv &= \frac{1}{1 - q}(q + (1 - q)F(t_2))\psi(\hat{v}) \\ &\quad + \int_{t_2}^{\bar{v}} \psi(v)f(v)dv - \left(F(a_2) - F(t_1)\right)\psi(\hat{v}). \end{aligned}$$

Substituting the above expression in (A.9) yields:

$$\begin{aligned}
& -\left(F(a_2) - F(t_1)\right)\psi(\hat{v}) - \frac{1}{1-q}(q + (1-q)F(t_2))\psi(\hat{v}) \\
& - \int_{t_2}^{\bar{v}} \psi(v)f(v)dv + \left(F(a_2) - F(t_1)\right)\psi(\hat{v}) \\
& - \left(1 - F(t_2)\right)\psi(\hat{v}) + \int_{t_2}^{\bar{v}} \psi(x)f(x)dx + (1-q)\frac{\partial t_2}{\partial q}\psi(t_2)f(t_2) \\
= & -\frac{1}{1-q}(q + (1-q)F(t_2))\psi(\hat{v}) - \left(1 - F(t_2)\right)\psi(\hat{v}) + (1-q)\frac{\partial t_2}{\partial q}\psi(t_2)f(t_2) < 0.
\end{aligned}$$

It then follows that increasing q makes \mathcal{M}^2 more desirable relative to \mathcal{M}^1 . ■

B Optimality of partitional measurements

B.1 Formal definition of the optimization problem

In this Appendix, we show how an optimal measurement in the models of Verrecchia (1983), Dye (1985) and Jung and Kwon (1988) simplify to a convex-partitional measurement with coarseness only at the marginal discloser. The proof involves many steps. For this reason, we first highlight the proof method.

Technically speaking, the problem under consideration is one of Bayesian persuasion, but with a constraint determined by the voluntary disclosure decision occurring after the design of the measurement system. Absent the issue of incentive compatibility for the firm's disclosure, the regulator problem boils down to the choice of a distribution of posterior means G such that the distribution of the asset value, F , is a mean-preserving spread of G . Due to convexity of the payoff function, the regulator would optimally select $G^* = F$, to maximize the ex ante expectation of firm value.

A complication arises when the firm has control over whether to disclose information, as the regulator must anticipate the consequences of the choice of G on the voluntary disclosure subgame. Namely, even though the firm (probabilistically) observes information generated by G , the market only sees the information that the firm elects to disclose. Therefore, the regulator's ex ante expectation of firm value should *not* be taken with respect to the measurement system G , but instead with respect to another distribution, denoted H_G , which is distribution of posterior means in the disclosure subgame (the subscript “ G ” in H_G represents

the fact that the subgame distribution of posteriors is a function of what measurement system the regulator imposes).

Specifically, we can compute H_G as follows. First, let

$$\mathcal{D}_G \equiv \{x \in \text{supp } G : \phi(x) - c \geq \phi(\mu(nd; G))\}$$

denote the disclosure region, that is, the realizations of X (in the support of G) such that the manager prefers disclosure over nondisclosure for a given nondisclosure expectation $\mu(nd; G)$. Further, let $\mathcal{ND}_G \equiv \text{supp } G \setminus \mathcal{D}_G$ denote the strategic nondisclosure region, that is, the complement of the disclosure region in the support of G . Given the disclosure and nondisclosure regions, the market's rational posterior expectation of the asset value conditional on nondisclosure is

$$\mu(nd; G) = \frac{q\mathbb{E}(V) + (1-q)\mathbb{P}(\mathcal{ND}_G)\mathbb{E}(X|X \in \mathcal{ND}_G)}{q + (1-q)\mathbb{P}(\mathcal{ND}_G)}. \quad (\text{B.11})$$

The expectation $\mu(nd; G)$ is a convex combination of the prior mean, $\mathbb{E}(V)$, and the expectation conditional on nondisclosure, $\mathbb{E}(X|X \in \mathcal{ND}_G)$. The weight on the conditional on the prior mean is the posterior probability that no verifiable information exists, given by the prior probability of no verifiable information, q , divided by the total probability of nonstrategic and strategic withholding, $q + (1-q)\mathbb{P}(\mathcal{ND}_G)$.

Second, note that if there exists a type x in the support of G that prefers disclosure over nondisclosure (i.e., such that $\phi(x) - c \geq \phi(\mu(nd; G))$), then all types $x' > x$ also prefer disclosure over nondisclosure. Thus, an equilibrium is characterized by a marginal discloser, denoted $\hat{x}(G)$, such that all types $x \geq \hat{x}(G)$ disclose and all $x < \hat{x}(G)$ withhold. If the disclosure region \mathcal{D}_G is empty, we set $\hat{x}(G) = \bar{v}$.

With this notation, the distribution of posterior means as is observed by the market is

$$H_G(x) = \begin{cases} 0 & \text{if } x \in [0, \mu(nd; G)) \\ q + (1-q)\mathbb{P}(\mathcal{ND}_G) & \text{if } x \in [\mu(nd; G), \hat{x}(G)) \\ q + (1-q)\mathbb{P}(X \leq x) & \text{if } x \in [\hat{x}(G), \bar{v}] \end{cases} \quad (\text{B.12})$$

²⁵By the law of iterated expectations, we can write $\mathbb{E}(X|X \in \mathcal{ND}_G)$ in place $\mathbb{E}(V|X \in \mathcal{ND}_G)$, as X is the posterior expectation of the asset value (i.e., $\mathbb{E}(V|X) = X$). Also, we write $\int_{\mathcal{ND}_G} x dG(x) = \mathbb{P}(\mathcal{ND}_G)\mathbb{E}(X|X \in \mathcal{ND}_G)$ because at an optimum the set of strategic withholders is nonempty as per Lemma B.4 in the Appendix.

The expression in (B.12) necessitates a few comments. Recall that H_G is a distribution function, and so it measures the probability that the posterior mean $X \leq x$. As such, H_G equals zero for all values less than the nondisclosure expectation, because none of these values (even if they are on the support of G) are disclosed in equilibrium. At $x = \mu(nd; G)$, the distribution has a discrete jump, because the nondisclosure expectation realizes when the firm has no verifiable information (with probability q) and when the firm is strategically withholding (with probability $(1 - q)\mathbb{P}(\mathcal{N}\mathcal{D}_G)$). The distribution then stays constant until x reaches the lowest value in the support of G such that disclosure is preferred over nondisclosure. From that point on, H_G (weakly) increases with x until the upper bound of the support, where H_G equals one because all informed lower types either withhold or disclose.

The following Lemma combines the previous discussion with Definitions 1 and 2 to provide a mathematical formulation of optimal measurements.

Lemma B.1 *A measurement G^* is optimal if it solves*

$$\begin{aligned} & \max_G \int_0^{\bar{v}} \phi''(x) S(x; H_G) dx \\ & \text{s.t.} \\ & [MPS] S(x; G) \leq S(x; F) \text{ for all } x \in [0, \bar{v}], \text{ with equality for } x = \bar{v} \\ & [BP] \mu(nd; G) \text{ is given by (B.11) and } H_G \text{ is given by (B.12),} \end{aligned}$$

where $S(x; G) \equiv \int_0^x G(y) dy$.

The objective function is obtained by integrating by parts twice the unconditional expectation of firm value taken with respect to the distribution H_G .²⁶ The label [MPS] for the first constraint stands for “mean-preserving spread”, as it states a necessary and sufficient condition for F to be a mean-preserving spread of G .²⁷ The second constraint is labeled [BP] as a mnemonics for “Bayesian plausibility”, and summarizes

²⁶The steps are as follows,

$$\int_0^{\bar{x}} \phi(x) dH_G(x) = \bar{v} - \int_0^{\bar{v}} \phi'(x) H_G(x) dx = \bar{v} - \phi'(\bar{v}) S(\bar{v}; H_G) + \int_0^{\bar{v}} \phi''(x) S(x; H_G) dx.$$

Then, we observe that the first two terms, $\bar{v} - \phi'(\bar{v}) S(\bar{v}; H_G)$, are independent of G , because H_G is a mean-preserving spread of F and so $S(\bar{v}; H_G) = S(\bar{v}; F)$. Hence, only the last term is relevant for optimization purposes.

²⁷Equality of the means is implied by the condition $S(\bar{v}; G) = S(\bar{v}; F)$. This fact can be seen by integrating by parts the unconditional expectation, $\int_0^{\bar{v}} x dG(x) = \bar{v} - S(\bar{v}; G)$.

the equilibrium of the disclosure subgame. We name this constraint as such because the unconditional expectation with respect to H_G equals the unconditional expectation of the asset value, $\mathbb{E}(V)$.

B.2 Characterization of the equilibrium in the disclosure subgame

Given our relatively unusual setting for the voluntary disclosure subgame, we say a few words on the equilibrium derivation when the posterior mean follows a generic distribution G .

Since the distribution G of posterior means might feature discrete jumps in correspondence with values that have positive probability mass, integrals in the analysis are to be interpreted as Lebesgue-Stieltjes integrals.²⁸ We specify whether the limits of integration are included or excluded, unless it makes no difference. We use $G(x_-)$ to denote the left limit of G at a point x . The right limit of G at a point x is simply $G(x)$, because distribution functions are right-continuous.

Let \hat{x} be the marginal discloser. The equilibrium nondisclosure price must equal

$$\begin{aligned} \mu(nd; G) &= \frac{q\mathbb{E}(V) + (1-q) \int_{[0, \hat{x})} x dG(x)}{q + (1-q)G(\hat{x}_-)} \\ &= \frac{q\mathbb{E}(V) + (1-q) [\hat{x}G(\hat{x}_-) - S(\hat{x}; G)]}{q + (1-q)G(\hat{x}_-)} \equiv \Upsilon(\hat{x}; G), \end{aligned} \tag{B.13}$$

where the second line uses integration by parts for distribution functions. The function $\Upsilon(\hat{x}; G)$ in (B.13) is the expectation of asset value conditional on nondisclosure and is obtained as follows: with probability q the firm has no verifiable information, so the expected asset value in this event is the unconditional mean $\mathbb{E}(V)$; with probability $(1-q)G(\hat{x}_-)$ the firm is withholding information, so the expected asset value is conditional on the event $X < \hat{x}$ (with strict inequality because the marginal discloser discloses). A marginal discloser \hat{x} is part of an equilibrium if $\phi(\hat{x}) - c \geq \phi(\Upsilon(\hat{x}; G))$ (implying that all types greater than \hat{x} disclose) and $\phi(x) - c < \phi(\Upsilon(\hat{x}; G))$ for all $x < \hat{x}$ in the support of G (implying that all types smaller than \hat{x} withhold).

If there are multiple equilibria in the voluntary disclosure subgame, we select the equilibrium with the highest ex ante expectation of firm value. Analogous equilibrium selection criteria are common in the disclosure literature (e.g., [Kamenica and Gentzkow, 2011](#); [Hart, Kremer and Perry, 2017](#); [Rappoport, 2017](#)).

Lemma B.2 below identifies the subgame equilibrium that maximizes the ex ante firm value as the one with

²⁸We cannot restrict the attention to only distribution of posterior means that admit a density because partitioned signal structures imply a distribution that is not absolutely continuous, since it features non-zero mass at certain points (e.g., all states within an interval yield the same posterior).

the lowest marginal discloser.

Lemma B.2 *Fix the distribution of posterior means G and the disclosure cost c . Let*

$$\mathcal{X}_+ \equiv \{x \in \text{supp } G : \phi(x) - c \geq \phi(\Upsilon(x; G))\}.$$

(i) *If \mathcal{X}_+ is empty, then nondisclosure by all types is the only equilibrium of the voluntary disclosure subgame.*

(ii) *If \mathcal{X}_+ is nonempty, then an equilibrium exists where the threshold type is $\hat{x}_c = \min \mathcal{X}_+$. Moreover, among all equilibria, ex ante firm value is maximal in this equilibrium.*

Proof of Lemma B.2. In part (i), the disclosure cost is so high that no type has an incentive to separate from the nondisclosing types (namely, the set \mathcal{X}_+ is empty). Hence, the unique equilibrium outcome is nondisclosure by all types.

In part (ii), some types have an incentive to separate. We prove part (ii) in a series of steps. Step 1 below argues that $\min \mathcal{X}_+$ exists. Step 2 shows that the threshold identified in this way is part of a subgame equilibrium for the case $c = 0$. Step 3 generalizes Step 2 to the case $c > 0$. Step 4 argues that in all other equilibria (which may or may not exist), ex ante firm value is lower than in the equilibrium with marginal discloser $\hat{x}_c = \min \mathcal{X}_+$.

Step 1. $\min \mathcal{X}_+$ exists.

Proof of Step 1. Suppose not. Then, for any $x \in \mathcal{X}_+$ there exists an $x' \in \mathcal{X}_+$ such that $x' < x$. Therefore, it is possible to construct a decreasing sequence $\{x_n\}$ in \mathcal{X}_+ . Such a sequence must converge to a limit because $x \geq 0$ implies that \mathcal{X}_+ is bounded from below. Call \hat{x} the limit of this sequence. For a minimum not to exist, we must have $\hat{x} \notin \mathcal{X}_+$. However, $\hat{x} \in \text{supp } G$ because the support of a random variable is a closed set. Also, we now argue that $\phi(\hat{x}) - c \geq \phi(\Upsilon(\hat{x}; G))$, which together with $\hat{x} \in \text{supp } G$ implies the contradiction that the limit $\hat{x} \in \mathcal{X}_+$. To show that the inequality holds in the limit, first observe that the left-hand side, $\phi(x_n) - c$, converges to $\phi(\hat{x}) - c$. Second, observe that $\Upsilon(x_n; G)$ converges to

$$\tilde{\Upsilon}(\hat{x}; G) \equiv \frac{q\mathbb{E}(V) + (1-q)[\hat{x}G(\hat{x}) - S(\hat{x}; G)]}{q + (1-q)G(\hat{x})} = \frac{q\mathbb{E}(V) + (1-q)\int_{[0, \hat{x}]} x dG(x)}{q + (1-q)G(\hat{x})},$$

which is the expectation conditional on the firm being uninformed or informed with $X \leq \hat{x}$.²⁹ The inclusion of $X = \hat{x}$ in the conditional expectation is what differentiates $\tilde{\Upsilon}(\hat{x}; G)$ from $\Upsilon(\hat{x}; G)$. Since inequalities are preserved in the limit, we have $\phi(\hat{x}) - c \geq \phi(\tilde{\Upsilon}(\hat{x}; G))$. From the latter inequality, we have $\hat{x} \geq \tilde{\Upsilon}(\hat{x}; G)$, which in turn implies $\tilde{\Upsilon}(\hat{x}; G) \geq \Upsilon(\hat{x}; G)$, as we are removing from the conditional expectation $\tilde{\Upsilon}(\hat{x}; G)$ a realization, \hat{x} , which is greater than the expectation itself.³⁰ Overall, $\phi(\hat{x}) - c \geq \phi(\tilde{\Upsilon}(\hat{x}; G)) \geq \phi(\Upsilon(\hat{x}; G))$ and so we reach the contradiction that $\hat{x} \in \mathcal{X}_+$.

Step 2. The threshold $\hat{x}_0 = \min \mathcal{X}_+$ is part of an equilibrium when $c = 0$.

Proof of Step 2. Since \hat{x}_0 (weakly) prefers disclosure over nondisclosure, all $x > \hat{x}_0$ strictly prefer disclosure. There remains to check that all types $x < \hat{x}_0$ prefer nondisclosure over disclosure. When $c = 0$, the condition $\phi(x) - c < \phi(\Upsilon(\hat{x}_0; G))$ is equivalent to $x < \Upsilon(\hat{x}_0; G)$ (because ϕ is strictly increasing). By contradiction, suppose that there existed a type $x < \hat{x}_0$ such that $x \geq \Upsilon(\hat{x}_0; G)$. Then, we would have $\Upsilon(x; G) \leq \Upsilon(\hat{x}_0; G)$, because we are removing from the conditional expectation $\Upsilon(\hat{x}_0; G)$ the types $[x, \hat{x}_0)$, which are greater than the expectation itself.³¹ As a consequence, $x \geq \Upsilon(\hat{x}_0; G) \geq \Upsilon(x; G)$, contradicting the fact that $x \notin \mathcal{X}_+$. We conclude that $x < \Upsilon(\hat{x}_0; G)$ for all types $x < \hat{x}_0$.

Step 3.1. $x < \Upsilon(x; G)$ for all $x < \hat{x}_0$ (if there are any such types).

Proof of Step 3.1. We know from the proof of Step 2 that $x < \Upsilon(\hat{x}_0; G)$ for all types $x < \hat{x}_0$. Hence, $\Upsilon(x; G) \geq \Upsilon(\hat{x}_0; G)$, because we are removing from the conditional expectation $\Upsilon(\hat{x}_0; G)$ the types in $[x, \hat{x}_0)$, which are lower than the expectation itself. So, $x < \Upsilon(\hat{x}_0; G) \leq \Upsilon(x; G)$.

Step 3.2. When $c > 0$, all types $x < \hat{x}_0$ (if there are any) prefer nondisclosure over disclosure. Further, $\hat{x}_c \geq \hat{x}_0$.

²⁹Convergence of $\Upsilon(x; G)$ to $\tilde{\Upsilon}(\hat{x}; G)$ as $x \downarrow \hat{x}$ follows from $G(\hat{x}) \leq G(x_-) \leq G(x)$ (because $x > \hat{x}$ and G is nondecreasing) and $G(x) \downarrow G(\hat{x})$ (because G is right-continuous). Hence, $\lim_{x \downarrow \hat{x}} G(x_-) = G(\hat{x})$.

³⁰Formally, if the point \hat{x} has no probability mass (i.e., $G(\hat{x}) - G(\hat{x}_-) = 0$), then adding the type $x = \hat{x}$ to the conditional expectation $\Upsilon(\hat{x}; G)$ is immaterial, and so $\tilde{\Upsilon}(\hat{x}; G) = \Upsilon(\hat{x}; G)$. Conversely, if there is probability mass on the point \hat{x} (i.e., $G(\hat{x}) - G(\hat{x}_-) > 0$), then

$$\tilde{\Upsilon}(\hat{x}; G) = \frac{[q + (1 - q)G(\hat{x}_-)] \Upsilon(\hat{x}; G) + (1 - q)[G(\hat{x}) - G(\hat{x}_-)] \hat{x}}{q + (1 - q)G(\hat{x})} \geq \Upsilon(\hat{x}; G)$$

if and only if $\hat{x} \geq \Upsilon(\hat{x}; G)$, as is the case.

³¹The latter claim follows from the fact that the inequality

$$\Upsilon(x; G) = \frac{[q + (1 - q)G(\hat{x}_0_-)] \Upsilon(\hat{x}_0; G) - (1 - q) \int_{[x, \hat{x}_0)} y dG(y)}{q + (1 - q)G(x_-)} \leq \Upsilon(\hat{x}_0; G)$$

boils down to $\int_{[x, \hat{x}_0)} [\Upsilon(\hat{x}_0; G) - y] dy \leq 0$, which holds because $y \geq \Upsilon(\hat{x}_0; G)$ for all $y \in [x, \hat{x}_0)$.

Proof of Step 3.2. From Step 3.1, we know that $x < \Upsilon(x; G)$ and so $\phi(x) - c < \phi(\Upsilon(x; G))$. Thus, $x \notin \mathcal{X}_+$ for all $x < \hat{x}_0$ and we must have $\hat{x}_c \geq \hat{x}_0$.

Step 3.3. $x > \Upsilon(x; G)$ for all $x > \hat{x}_0$ (if there are any such types).

Proof of Step 3.3. $\Upsilon(x; G)$ is obtained by adding to the conditional expectation $\Upsilon(\hat{x}_0; G)$, which is smaller than x , the types in $[\hat{x}_0, x)$, which are also smaller than x . In particular, we have

$$\Upsilon(x; G) = \frac{[q + (1 - q)G(\hat{x}_{0-})] \Upsilon(\hat{x}_0; G) + (1 - q) \int_{[\hat{x}_0, x)} y dG(y)}{q + (1 - q)G(x_-)} < x$$

if and only if

$$[q + (1 - q)G(\hat{x}_{0-})] [x - \Upsilon(\hat{x}_0)] + (1 - q) \int_{[\hat{x}_0, x)} (x - y) dG(y) > 0.$$

The inequality above holds because $x > \hat{x}_0 \geq \Upsilon(\hat{x}_0; G)$.

Step 3.4. The threshold $\hat{x}_c = \min \mathcal{X}_+$ is part of an equilibrium when $c > 0$.

Proof of Step 3.4. By Steps 3.1 and 3.2, it suffices to check that all types $x \in [\hat{x}_0, \hat{x}_c)$ (if there are any) prefer nondisclosure over disclosure. These types withhold because $\phi(x) - c < \phi(\Upsilon(x; G)) \leq \phi(\Upsilon(\hat{x}_c; G))$. The first inequality follows from $x \notin \mathcal{X}_+$. The second inequality from Step 3.3, because $\Upsilon(\hat{x}_c; G)$ is obtained by adding to the conditional expectation $\Upsilon(x; G)$ the types $[x, \hat{x}_c)$, which are greater than the expectation itself. Therefore, $\Upsilon(\hat{x}_c; G) \geq \Upsilon(x; G)$.

Step 4. Across all equilibria, the greatest ex ante firm value is achieved in the equilibrium with marginal discloser \hat{x}_c .

Proof of Step 4. Suppose that there is another equilibrium with threshold $\hat{x} > \hat{x}_c$. Consider the event $X \geq \hat{x}$. The probability of this event, $\int_{[\hat{x}, \bar{v}]} dH(x) = (1 - q)[1 - G(\hat{x}_-)]$, and the distribution of $X|X \geq \hat{x}$ are the same in both equilibria. These facts have two consequences. First, the expected firm value conditional on this event, $\int_{[\hat{x}, \bar{v}]} \phi(x) d\frac{H(x)}{(1 - q)[1 - G(\hat{x}_-)]}$, is the same in both equilibria. Second, the expected asset value conditional on the complement event, $\int_{[0, \hat{x}]} x d\frac{H(x)}{q + (1 - q)G(\hat{x}_-)}$, is the same in both equilibria.³² The distribution of $X|X < \hat{x}$ in the equilibrium with threshold \hat{x}_c is a mean-preserving spread of the distribution of $X|X < \hat{x}$ in the equilibrium with threshold \hat{x} , because the latter distribution is degenerate at the nondisclosure expectation (all types $x < \hat{x}$ withhold in that equilibrium). Since firm value is convex in the

³²The claim exploits the law of iterated expectations, by which $\mathbb{E}(X|X < \hat{x}) = [\mathbb{E}(V) - \mathbb{P}(X \geq \hat{x})\mathbb{E}(X|X \geq \hat{x})]/[1 - \mathbb{P}(X \geq \hat{x})]$.

posterior mean, the ex ante expected firm value is greater in the equilibrium with threshold \hat{x}_c than in the one with threshold \hat{x} . ■

B.3 A partitional representation

Next, we reduce the complexity of the choice by showing the optimality of a single imprecise interval $[t, a_2]$. That is, if $V \notin [t, a_2]$, then the manager can credibly convey the exact realization of the asset value (i.e., the verifiable signal is $X = V$). By contrast, if $V \in [t, a_2]$, then the manager can credibly convey only the fact that the realized asset value belongs to that interval (i.e., the verifiable signal is $X = \mathbb{E}[V|t \leq V \leq a_2]$).

Proposition 5 *Fix the disclosure cost $c \geq 0$. If G^* is an optimal measurement, then there exist $t^* \leq \hat{x}^* \leq a_2^*$ such that:*

- (i) $\hat{x}^* = \mathbb{E}[V|t^* \leq V \leq a_2^*]$;
- (ii) $\mu(nd; G^*) = \frac{q\mathbb{E}(V) + (1-q)F(t^*)\mathbb{E}[V|V < t^*]}{q + (1-q)F(t^*)}$;
- (iii) $\phi(\hat{x}^*) - c = \phi(\mu(nd; G^*))$; and
- (iv) *the equilibrium distribution of posterior means in the disclosure subgame is*

$$H_{G^*}(x) = \begin{cases} 0 & \text{if } x \in [0, \mu(nd; G^*)) \\ q + (1-q)F(t^*) & \text{if } x \in [\mu(nd; G^*), \hat{x}^*) \\ q + (1-q)F(a_2^*) & \text{if } x \in [\hat{x}^*, a_2^*] \\ q + (1-q)F(x) & \text{if } x \in (a_2^*, \bar{v}] \end{cases} \quad .^{33} \quad (\text{B.14})$$

The distribution of posterior means in (B.14) is determined as follows. Starting from (B.12), plug in the specific values for $\mathbb{P}(\mathcal{N}\mathcal{D}_{G^*})$ and $\mathbb{P}(X \leq x)$ that apply in the case with an imprecise interval. First, we have that $\mathbb{P}(\mathcal{N}\mathcal{D}_{G^*}) = F(t^*)$, because the probability of strategic withholding is the probability of the

³³As for the costless disclosure case of Proposition 1, note how Proposition 5 does not specify what the measurement system should be for realizations $x < t^*$. The reason is that these values are withheld in equilibrium and, consequently, they do not belong to the support of the distribution of posterior means as are observed by the market, H_{G^*} . Hence, there are multiple optimal measurements that agree on $x \geq t^*$ but differ on $x < t^*$. For example, an optimal measurement can feature either perfect measurement of $x < t^*$ or a mass point at $\mathbb{E}(V|V < t^*)$.

realized value being less than the lower bound of the imprecise interval. Second, $\mathbb{P}(X \leq x) = F(a_2^*)$ for all $x \in [\hat{x}^*, a_2^*]$, because all realized values in $[t^*, a_2^*]$ are pooled together. Last, $\mathbb{P}(X \leq x) = F(x)$ for $x \in (a_2^*, \bar{v}]$, because above the upper bound of the imprecise interval the measurement is precise and the sender discloses his signal.

B.3.1 Preliminary observations

We prove this Proposition relying on several lemmas. Let G^* be an optimal measurement. An initial observation is that G^* must induce disclosure by a positive mass of types. This statement is formalized in the following Lemma B.3.

Lemma B.3 *At the optimum, the disclosure region, \mathcal{D}_{G^*} , is nonempty. Therefore, part (ii) of Lemma B.2 applies and the marginal discloser is $\hat{x}^* = \min \mathcal{X}_+$.*

Proof of Lemma B.3. If G^* were such that (almost) all informed types chose nondisclosure, the distribution of posterior means H_{G^*} would be degenerate at $x = \mathbb{E}(V)$. By contrast, if the regulator chose perfect measurement (i.e., $G = F$), types close to the upper bound \bar{v} would disclose, because $\phi(\bar{v}) - c > \phi(\mathbb{E}(V))$ (our working assumption throughout) and $\phi(\mathbb{E}(V)) \geq \phi(\mu(nd; F))$ (as $\mu(nd; F)$ is a convex combination between the prior mean, $\mathbb{E}(V)$, and an expectation truncated from above). Therefore, the equilibrium distribution of posterior means under perfect measurement, H_F , would be nondegenerate and, as such, would be a mean-preserving spread of H_{G^*} . Since the objective function is convex, the ex ante expected firm value is strictly greater under H_F than under H_{G^*} , so G^* cannot be an optimal measurement. ■

A second observation is that G^* must induce strategic withholding by a positive mass of types, as stated by Lemma B.4 below.

Lemma B.4 *At the optimum, the strategic withholding region, $\mathcal{N}\mathcal{D}_{G^*}$, has positive mass. Therefore, $\int_{\mathcal{N}\mathcal{D}_{G^*}} x dG^*(x) = \mathbb{P}(\mathcal{N}\mathcal{D}_{G^*})\mathbb{E}(X|X \in \mathcal{N}\mathcal{D}_{G^*})$.*

Proof of Lemma B.4. By contradiction, suppose that $\mathbb{P}(\mathcal{N}\mathcal{D}_{G^*}) = 0$. Then, $\mu(nd; G^*) = \mathbb{E}(V)$. Since almost all types disclose, $\phi(x) - c \geq \phi(\mathbb{E}(V))$, and thus also $x \geq \mathbb{E}(V)$, for almost all x . By the law of iterated expectations, G^* , and so also H_{G^*} , are degenerate at $x = \mathbb{E}(V)$. As we have seen in the proof of Lemma B.3, the regulator would strictly prefer perfect measurement $G = F$ to G^* , so G^* is not optimal. ■

A third observation is that there must exist a $t = \max\{x \in \mathcal{V} : x \leq \hat{x}^* \text{ and } S(x; G^*) = S(x; F)\}$, because this set is compact (as $S(x; G)$ is continuous in x) and nonempty (as $S(0; G^*) = S(0; F)$). Also, there must exist an $a_2 = \min\{x \in \mathcal{V} : x \geq \hat{x}^* \text{ and } S(x; G^*) = S(x; F)\}$, because this set is compact and nonempty (as $S(\bar{v}; G^*) = S(\bar{v}; F)$ by the [MPS] constraint). The following Lemma B.5 describes some useful properties of t and a_2 that are implied by the equality $S(x; G^*) = S(x; F)$. We know from Lemma 3 in Dworzak and Martini (2018) that such x satisfies $G(x) = F(x)$. On top of that, we show that G must be continuous at x (i.e., $G(x_-) = G(x)$).

Lemma B.5 *Suppose that the distribution F is a mean-preserving spread of G . If $x \in \mathcal{V}$ is such that $S(x; G) = S(x; F)$, then $G(x_-) = G(x) = F(x)$.*

Proof of Lemma B.5. If $x = \bar{v}$, then because both F and G are distributions we must have $G(\bar{v}) = F(\bar{v}) = 1$. So, suppose for the remainder of the proof that $x \in [0, \bar{v})$. We make two observations:

- First, we argue that if $x \in [0, \bar{v})$, then $G(x) \leq F(x)$. By contradiction, suppose that $G(x) > F(x)$. For $z > x$, $S(z; G) = S(x; G) + \int_x^z G(y)dy$ and $S(z; F) = S(x; F) + \int_x^z F(y)dy$. Since $S(x; G) = S(x; F)$, $S(z; F) - S(z; G) = \int_x^z [F(y) - G(y)]dy$. Because F and G are right-continuous and $G(x) > F(x)$, there exists a $z > x$ such that $S(z; F) - S(z; G) < 0$, contradicting the fact that F is a mean-preserving spread of G . Thus, it must be $G(x) \leq F(x)$.
- Second, we argue that if $x \in (0, \bar{v})$, then $G(x_-) \geq F(x)$. For $z < x$, $S(z; G) = S(x; G) - \int_z^x G(y)dy$ and $S(z; F) = S(x; F) - \int_z^x F(y)dy$. Since $S(x; G) = S(x; F)$, $S(z; F) - S(z; G) = \int_z^x [G(y) - F(y)]dy$. Suppose, by contradiction, that $G(x_-) < F(x)$. Then, by continuity of F there would exist a $z < x$ such that $S(z; F) - S(z; G) < 0$, contradicting the fact that F is a mean-preserving spread of G . Thus, it must be $G(x_-) \geq F(x)$.

If $x = 0$, $F(0) = 0$, by the first observation $0 \leq G(0) \leq F(0) = 0$, hence $G(0) = 0 = F(0)$ and the proof is concluded. If $x > 0$, by the first observation we have $G(x) \leq F(x)$, while by the second observation we have $G(x) \geq G(x_-) \geq F(x)$. Hence, $G(x) = G(x_-) = F(x)$. ■

For future reference, note that using the notation $\mathbb{P}(\mathcal{N}\mathcal{D}_{G^*}) = G^*(\hat{x}_-^*)$ and $\mathbb{P}(X \leq x) = G(x)$, the equilibrium distribution of posterior means in (B.12) can be rewritten as

$$H_{G^*}(x) = \begin{cases} 0 & \text{if } x \in [0, \mu(nd; G)) \\ q + (1 - q)G^*(\hat{x}_-^*) & \text{if } x \in [\mu(nd; G), \hat{x}^*) \\ q + (1 - q)G^*(x) & \text{if } x \in [\hat{x}^*, \bar{v}] \end{cases} \quad (\text{B.15})$$

B.3.2 Overview of the proof of Proposition 5

The objective is to show that G^* is a partitional measurement with imprecise interval $[t, a_2]$, where the end points of this interval are defined as above. We summarize below the steps involved in the proof:

Step 1. Lemma B.6 shows that the optimal measurement must involve perfect measurement of all $V > a_2$.

Equivalently, the distributions of the signal $X|X > a_2$ and of the asset value $V|V > a_2$ must coincide.

Step 2. We argue that, of all the values in the interval $[t, a_2]$, only $x = \mathbb{E}(V|t \leq V \leq a_2)$ is in the support of the optimal measurement. Namely:

Step 2.1. Lemma B.7 shows that $G^*(\hat{x}^*) = G^*(a_2) = F(a_2)$, so the c.d.f. G^* is flat on $[\hat{x}^*, a_2]$;

Step 2.2. Lemma B.8 shows that $G^*(\hat{x}_-^*) = G^*(t) = F(t)$,³⁴ so G^* is flat on $[t, \hat{x}^*)$;

Step 2.3. Lemma B.9 shows that the marginal discloser is $\hat{x}^* = \mathbb{E}(V|t \leq V \leq a_2)$.

Step 3. Lemma B.10 shows that the marginal discloser must be exactly indifferent between disclosure and nondisclosure.

B.3.3 The lemmas that imply Proposition 5

At this point, we proceed to state and prove the lemmas mentioned above.

Lemma B.6 $G^*(x) = F(x)$ for all $x \in [a_2, \bar{v}]$.

³⁴We use $G(x_-)$ to denote the left limit of G at point x .

Proof of Lemma B.6. If $a_2 = \bar{v}$, then $G^*(a_2) = F(a_2) = 1$ and the proof is concluded. So, suppose instead that $a_2 < \bar{v}$. By Lemma B.5, if F is a mean-preserving spread of G and $S(x; G) = S(x; F)$ for some x , then $G(x) = F(x)$. Thus, we have $G^*(a_2) = F(a_2)$ and there remains to verify the claim on the lemma for $x \in (a_2, \bar{v}]$.

Recall that we denote by H_G the c.d.f. of the posterior mean when the measurement system is G . The [BP] constraint and (B.15) imply that $[q + (1 - q)G(\hat{x}_-)]\mu(nd; G) = q\mathbb{E}(V) + (1 - q)[\hat{x}G(\hat{x}_-) - S(\hat{x}; G)]$. Hence, the integral of the c.d.f. H_G takes the form

$$S(x; H_G) = \begin{cases} 0 & \text{if } x \in [0, \mu(nd; G)) \\ q(x - \mathbb{E}(V)) + (1 - q)[(x - \hat{x})G(\hat{x}_-) + S(\hat{x}; G)] & \text{if } x \in [\mu(nd; G), \hat{x}) \\ q(x - \mathbb{E}(V)) + (1 - q)S(x; G) & \text{if } x \in [\hat{x}, \bar{v}] \end{cases} \quad (\text{B.16})$$

Further, by Lemma 5 and (B.16), the value of the objective function in equilibrium is

$$\int_0^{\bar{v}} \phi''(x)S(x; H_{G^*})dx = \int_0^{a_2} \phi''(x)S(x; H_{G^*})dx + \int_{a_2}^{\bar{v}} \phi''(x)[q(x - \mathbb{E}(V)) + (1 - q)S(x; G)]dx. \quad (\text{B.17})$$

The [MPS] constraint imposes the inequality $S(x; G^*) \leq S(x; F)$. To maximize the second term in the right-hand side of (B.17), the distributions of $X|X > a_2$ and $V|V > a_2$ must be the same, so that $S(x; G) = S(x; F)$ for all $x \in (a_2, \bar{v}]$. ■

Lemma B.7 $G^*(\hat{x}^*) = F(a_2)$.

Proof of Lemma B.7. If $a_2 = \hat{x}^*$, then $G^*(\hat{x}^*) = G^*(a_2) = F(a_2)$ (the second equality by Lemma B.6), which concludes the proof. If instead $a_2 > \hat{x}^*$, suppose by contradiction that $F(a_2) > G^*(\hat{x}^*)$. Then, let $\underline{a}_2 \equiv \max\{\hat{x}^*, F^{-1}(G^*(\hat{x}^*))\}$. We argue that the following measurement system strictly improves the ex ante firm value,

$$G^{**}(x) = \begin{cases} G^*(x) & \text{if } x \in [0, \hat{x}^*) \\ F(a'_2) & \text{if } x \in [\hat{x}^*, a'_2] \\ F(x) & \text{if } x \in (a'_2, \bar{v}] \end{cases}$$

where $a'_2 \in (\underline{a}_2, a_2)$ solves

$$S(a'_2; G^{**}) = S(a'_2; F). \quad (\text{B.18})$$

The condition $a'_2 > \underline{a}_2$ guarantees that both $a'_2 > \hat{x}^*$ and $F(a'_2) > G^*(\hat{x}^*)$, so that the c.d.f. G^{**} is nondecreasing. Also, note that $S(a'_2; G^{**}) = S(\hat{x}^*; G^*) + \int_{\hat{x}^*}^{a'_2} F(a'_2) dx$. Existence of a solution a'_2 to (B.18) is established as follows:

- If $\underline{a}_2 = \hat{x}^*$, then (B.18) evaluated at $a'_2 = \underline{a}_2$ becomes $S(\hat{x}^*; G^*) < S(\hat{x}^*; F)$ by the definition of a_2 being the smallest $x \geq \hat{x}^*$ such that $S(x; G^*) = S(x; F)$. If $\underline{a}_2 = F^{-1}(G^*(\hat{x}^*))$, then (B.18) evaluated at $a'_2 = \underline{a}_2$ becomes $S(\hat{x}^*; G^*) + \int_{\hat{x}^*}^{\underline{a}_2} G^*(\hat{x}^*) dx < S(\underline{a}_2; F)$, because $S(\hat{x}^*; G^*) + \int_{\hat{x}^*}^{\underline{a}_2} G^*(\hat{x}^*) dx \leq S(\hat{x}^*; G^*) + \int_{\hat{x}^*}^{\underline{a}_2} G^*(x) dx = S(\underline{a}_2; G^*)$ (as G^* is nondecreasing) and $S(\underline{a}_2; G^*) < S(\underline{a}_2; F)$ (by the definition of a_2). In either case, when evaluated at $a'_2 = \underline{a}_2$, the left-hand side of (B.18) is strictly smaller than the right-hand side.
- By contrast, when evaluated at $a'_2 = a_2$, equation (B.18) becomes $S(\hat{x}^*; G^*) + \int_{\hat{x}^*}^{a_2} F(a_2) dx > S(a_2; F)$, because $S(\hat{x}^*; G^*) + \int_{\hat{x}^*}^{a_2} F(a_2) dx = S(a_2; G^*) + \int_{\hat{x}^*}^{a_2} [F(a_2) - G^*(x)] dx > S(a_2; G^*)$ (as $G^*(x) \leq F(a_2)$ for $x \leq a_2$, with strict inequality in a neighborhood to the right of \hat{x}^* due to $G^*(\hat{x}^*) < F(a_2)$ and G^* being right-continuous) and $S(a_2; G^*) = S(a_2; F)$ (by the definition of a_2). Therefore, when evaluated at $a'_2 = a_2$, the left-hand side of (B.18) is strictly larger than the right-hand side.
- Continuity implies existence of a $a'_2 \in (\underline{a}_2, a_2)$ satisfying (B.18).

We need to verify that G^{**} satisfies the [MPS] condition, so that it is a feasible measurement system:

- For $x \in [0, \hat{x}^*]$, G^* and G^{**} coincide, so $S(x; G^{**}) = S(x; G^*) \leq S(x; F)$, because G^* satisfies [MPS].
- For $x \in (\hat{x}^*, a'_2)$, we have $S(x; G^{**}) = S(a'_2; G^{**}) - \int_x^{a'_2} F(a'_2) dy \leq S(a'_2; F) - \int_x^{a'_2} F(y) dy = S(x; F)$ if and only if $\int_x^{a'_2} [F(y) - F(a'_2)] dy \leq 0$ (because $S(a'_2; G^{**}) = S(a'_2; F)$ by definition of a'_2). The inequality holds because F is increasing.
- For $x \in [a'_2, \bar{v}]$, $S(x; G^{**}) = S(a'_2; G^{**}) + \int_{a'_2}^x F(y) dy = S(a'_2; F) + \int_{a'_2}^x F(y) dy = S(x; F)$. In particular, $S(\bar{v}; G^{**}) = S(\bar{v}; F)$, which implies equality of means.

Finally, we show that the objective function in Lemma 5 is strictly greater when evaluated at G^{**} than at G^* . For this purpose, it suffices to show that $S(x; H_{G^*}) \leq S(x; H_{G^{**}})$ for all $x \in \mathcal{V}$, with strict inequality

on a subset of positive Lebesgue measure.

- For $x \in [0, \hat{x}^*]$, G^* and G^{**} coincide. Hence, by equation (B.13) we have $\mu(nd; G^*) = \mu(nd; G^{**})$ and by equation (B.16) we have $S(x; H_{G^*}) = S(x; H_{G^{**}})$.
- For all $x > \hat{x}^*$ we have $S(x; H_G) = q(x - \mathbb{E}(V)) + (1 - q)S(x; G)$ by equation (B.13). Therefore, the inequality to check is $S(x; G^{**}) \geq S(x; G^*)$.
- For $x \in (\hat{x}^*, a'_2)$, $S(x; G^{**}) - S(x; G^*) = \int_{\hat{x}^*}^x [F(a'_2) - G^*(y)]dy$. This difference is nonnegative if $G^*(x) \leq F(a'_2)$. If instead $G^*(x) > F(a'_2)$, then $G^*(y) > F(a'_2)$ for all $y > x$ and so $S(x; G^{**}) - S(x; G^*) \geq S(a'_2; G^{**}) - S(a'_2; G^*)$. By the definition of a'_2 , $S(a'_2; G^{**}) - S(a'_2; G^*) = S(a'_2; F) - S(a'_2; G^*)$. By the definition of a_2 , $S(a'_2; F) - S(a'_2; G^*) > 0$. Hence, $S(x; G^{**}) \geq S(x; G^*)$ for all $x \in (\hat{x}^*, a'_2)$.
- For $x \in [a'_2, a_2)$, $S(x; G^{**}) = S(x; F) > S(x; G^*)$.
- For $x \in [a_2, \bar{v}]$, by Lemma B.6 we have $S(x; G^{**}) = S(x; G^*) = S(x; F)$.

Overall, we have shown that if $F(a_2) > G^*(\hat{x}^*)$, then there is another measurement system, G^{**} , that satisfies [MPS] and strictly increases ex ante firm value relative to G^* . Therefore, such a G^* cannot be optimal. ■

Lemma B.8 $G^*(\hat{x}_-) = G^*(t) = F(t)$.

Proof of Lemma B.8. Lemma B.5 implies $G^*(t) = F(t)$. If $t = \hat{x}^*$, Lemma B.5 also directly implies the claim $G^*(t) = G^*(\hat{x}_-)$. Suppose instead that $t < \hat{x}^*$. By way of contradiction, further suppose that $F(t) < G^*(\hat{x}_-)$. Then, let $\bar{t} \equiv \min\{\hat{x}^*, F^{-1}(G^*(\hat{x}_-))\}$. We now show that the following measurement system strictly increases the ex ante firm value,

$$G^{**}(x) = \begin{cases} F(x) & \text{if } x \in [0, t') \\ F(t') & \text{if } x \in [t', \hat{x}^*) \\ G^*(x) & \text{if } x \in [\hat{x}^*, \bar{v}] \end{cases},$$

where $t' \in (t, \bar{t})$ solves

$$S(\hat{x}^*; G^{**}) = S(\hat{x}^*; G^*). \tag{B.19}$$

The condition $t' < \bar{t}$ guarantees that both $t' < \hat{x}^*$ and $F(t') < G^*(\hat{x}_-^*)$, so that the c.d.f. G^{**} is nondecreasing. Also, note that $S(\hat{x}^*; G^{**}) = S(t'; F) + \int_{t'}^{\hat{x}^*} F(t')dx$. Existence of a solution t' to (B.19) is established as follows:

- When evaluated at $t' = t$, equation (B.19) becomes $S(t; F) + \int_t^{\hat{x}^*} F(t)dx < S(\hat{x}^*; G^*)$, because $S(t; F) + \int_t^{\hat{x}^*} F(t)dx = S(t; G^*) + \int_t^{\hat{x}^*} F(t)dx$ (by the definition of t being such that $S(t; F) = S(t; G^*)$) and $S(t; G^*) + \int_t^{\hat{x}^*} F(t)dx < S(\hat{x}^*; G^*) = S(t; G^*) + \int_t^{\hat{x}^*} G^*(x)dx$. The latter inequality holds if and only if $\int_t^{\hat{x}^*} [G^*(x) - F(t)]dx > 0$, which is satisfied because $G^*(x) \geq G^*(t) = F(t)$ for $x > t$ (by Lemma B.5), with strict equality in a neighborhood to the left of \hat{x}^* (because we are assuming, by contradiction, that $F(t) < G^*(\hat{x}_-^*)$).
- If $\bar{t} = \hat{x}^*$, then (B.19) evaluated at $t' = \bar{t}$ becomes $S(\hat{x}^*; F) > S(\hat{x}^*; G^*)$ by the definition of t being the greatest $x \leq \hat{x}^*$ such that $S(x; F) = S(x; G^*)$. If $\bar{t} = F^{-1}(G^*(\hat{x}_-^*))$, then (B.19) evaluated at $t' = \bar{t}$ becomes $S(\bar{t}; F) + \int_{\bar{t}}^{\hat{x}^*} G^*(\hat{x}_-^*)dx > S(\hat{x}^*; G^*)$, because $S(\bar{t}; F) + \int_{\bar{t}}^{\hat{x}^*} G^*(\hat{x}_-^*)dx \geq S(\bar{t}; F) + \int_{\bar{t}}^{\hat{x}^*} G^*(x)dx$ (because G^* is nondecreasing) and $S(\bar{t}; F) > S(\hat{x}^*; G^*) - \int_{\bar{t}}^{\hat{x}^*} G^*(x)dx = S(\bar{t}; G^*)$ (by the definition of t). In either case, when evaluated at $t' = \bar{t}$, the left-hand side of (B.19) is strictly greater than the right-hand side.
- Continuity implies existence of a $t' \in (t, \bar{t})$ satisfying (B.19).

We need to verify that G^{**} satisfies the [MPS] condition, so that it is a feasible measurement system:

- For $x \in [0, t']$, G^{**} and F coincide, so $S(x; G^{**}) = S(x; F)$.
- For $x \in (t', \hat{x}^*)$, we have $S(x; G^{**}) = S(t'; G^{**}) + \int_{t'}^x F(t')dy \leq S(t; F) + \int_{t'}^x F(y)dy = S(x; F)$ if and only if $\int_{t'}^x [F(t') - F(y)]dy \leq 0$ (because $S(t'; G^{**}) = S(t'; F)$). The inequality holds because F is increasing.
- For $x \in [\hat{x}^*, \bar{v}]$, $S(x; G^{**}) = S(\hat{x}^*; G^{**}) + \int_{\hat{x}^*}^x G^*(y)dy = S(\hat{x}^*; G^*) + \int_{\hat{x}^*}^x G^*(y)dy = S(x; G^*)$, because $S(\hat{x}^*; G^*) = S(\hat{x}^*; G^{**})$ by (B.19). We have $S(x; G^{**}) = S(x; G^*) \leq S(x; F)$ because G^* satisfies [MPS].
- Equality of the means is satisfied because $S(\bar{v}; G^{**}) = S(\bar{v}; G^*)$, and G^* has the same mean as F .

Last, we show that the objective function in Lemma 5 is strictly greater when evaluated at G^{**} than at G^* . We consider separately two cases: when $\phi(t') - c < \phi(\Upsilon(t'; F))$ and when $\phi(t') - c \geq \phi(\Upsilon(t'; F))$.

Proof for Case $\phi(t') - c < \phi(\Upsilon(t'; F))$. In this case, under the measurement G^{**} there exists an equilibrium with threshold \hat{x}^* and nondisclosure expectation

$$\begin{aligned}\mu(nd; G^{**}) &= \frac{q\mathbb{E}(V) + (1-q)[\hat{x}^*G^{**}(\hat{x}_-) - S(\hat{x}^*; G^{**})]}{q + (1-q)G^{**}(\hat{x}_-)} \\ &= \frac{q\mathbb{E}(V) + (1-q)[\hat{x}^*F(t') - S(t'; F) - (\hat{x}^* - t')F(t')]}{q + (1-q)F(t')} = \Upsilon(t'; F),\end{aligned}$$

where the second equality uses the definition of G^{**} . This combination of marginal discloser and nondisclosure expectation constitutes an equilibrium of the disclosure subgame for the following reasons:

- Notice that $\Upsilon(t'; F) \leq \mu(nd; G^*)$, because

$$\begin{aligned}\Upsilon(t'; F) &= \frac{q\mathbb{E}(V) + (1-q)[\hat{x}^*F(t') - S(\hat{x}^*; G^*)]}{q + (1-q)F(t')} \\ &\leq \frac{q\mathbb{E}(V) + (1-q)[\hat{x}^*G^*(\hat{x}_-) - S(\hat{x}^*; G^*)]}{q + (1-q)G^*(\hat{x}_-)} = \mu(nd; G^*),\end{aligned}$$

where the equality follows from (B.19) and the inequality from $F(t') < G^*(\hat{x}_-)$.³⁵

- Types $x \geq \hat{x}^*$ strictly prefers disclosure, because $\phi(\hat{x}^*) - c \geq \phi(\mu(nd; G^*))$ (as \hat{x}^* weakly preferred disclosure under measurement G^*) and $\mu(nd; G^*) \geq \Upsilon(t'; F)$ (from the previous observation).

- Values $x \in (t', \hat{x}^*)$ are not in the support of G^{**} .

- Types $x \leq t'$ strictly prefer nondisclosure by the assumption for this case.

For future reference, note that in this equilibrium the distribution of posterior means from the market's perspective is

$$\hat{H}_{G^{**}}(x) \equiv \begin{cases} 0 & \text{if } x \in [0, \Upsilon(t'; F)) \\ q + (1-q)F(t') & \text{if } x \in [\Upsilon(t'; F), \hat{x}^*) \\ q + (1-q)G^*(x) & \text{if } x \in [\hat{x}^*, \bar{v}] \end{cases} \quad (\text{B.20})$$

Using (B.16), we compare the ex ante firm value between the regulator preferred equilibrium under G^*

³⁵Namely, when $F(t') < G^*(\hat{x}_-)$, $\Upsilon(t'; F) \leq \mu(nd; G)$ if and only if $q(\mathbb{E}(V) - \hat{x}^*) - (1-q)S(\hat{x}^*; G^*) \leq 0$, which holds if $\hat{x}^* \geq \mu(nd; G^*)$. The last inequality is implied by $\phi(\hat{x}^*) - c \geq \phi(\mu(nd; G^*))$.

and the equilibrium above under G^{**} ,

$$\begin{aligned}
\int_0^{\bar{v}} \phi''(x) S(x; \hat{H}_{G^{**}}) dx &= \int_{\Upsilon(t'; F)}^{\hat{x}^*} \phi''(x) \{q(x - \mathbb{E}(V)) + (1 - q)[(x - \hat{x}^*)F(t') + S(\hat{x}^*; G^*)]\} dx \\
&+ \int_{\hat{x}^*}^{\bar{v}} \phi''(x) \{q(x - \mathbb{E}(V)) + (1 - q)S(x; G^*)\} dx \\
&\geq \int_{\mu(nd; G^*)}^{\hat{x}^*} \phi''(x) \{q(x - \mathbb{E}(V)) + (1 - q)[(x - \hat{x}^*)G^*(\hat{x}_-) + S(\hat{x}^*; G^*)]\} dx \\
&+ \int_{\hat{x}^*}^{\bar{v}} \phi''(x) \{q(x - \mathbb{E}(V)) + (1 - q)S(x; G^*)\} dx = \int_0^{\bar{v}} \phi''(x) S(x; H_{G^*}) dx,
\end{aligned}$$

where the inequality follows from $\Upsilon(t'; F) \leq \mu(nd; G^*)$ and $F(t') < G^*(\hat{x}_-)$. Ex ante firm value is thus greater under G^{**} than under G^* .

The equilibrium here identified under measurement G^{**} is not necessarily the one where ex ante firm value is maximal. However, we have shown that under G^{**} there exists an equilibrium with greater ex ante firm value than the regulator preferred equilibrium under G^* . Therefore, in the regulator preferred equilibrium under G^{**} ex ante firm value is at least as large.

Proof for Case $\phi(t') - c \geq \phi(\Upsilon(t'; F))$. In this case, we have $t' \geq \hat{x}(F)$. Therefore, under G^{**} the regulator preferred subgame equilibrium has threshold $\hat{x}^{**} = \hat{x}(F)$. In this equilibrium, the distribution of posterior means from the market's perspective is

$$H_{G^{**}}(x) = \begin{cases} 0 & \text{if } x \in [0, \Upsilon(\hat{x}^{**}; F)) \\ q + (1 - q)F(\hat{x}^{**}) & \text{if } x \in [\Upsilon(\hat{x}^{**}; F), \hat{x}^{**}) \\ q + (1 - q)F(x) & \text{if } x \in [\hat{x}^{**}, t') \\ q + (1 - q)F(t') & \text{if } x \in [t', \hat{x}^*) \\ q + (1 - q)G^*(x) & \text{if } x \in [\hat{x}^*, \bar{v}] \end{cases}. \quad (\text{B.21})$$

In the previous case, we have shown that $\int_0^{\bar{v}} \phi''(x) S(x; \hat{H}_{G^{**}}) dx \geq \int_0^{\bar{v}} \phi''(x) S(x; H_{G^*}) dx$. To complete the proof for this case, we now argue that $\int_0^{\bar{v}} \phi''(x) S(x; H_{G^{**}}) dx \geq \int_0^{\bar{v}} \phi''(x) S(x; \hat{H}_{G^{**}}) dx$. From (B.20) and (B.21), we see that the probability $\mathbb{P}(X \geq \hat{x}^*) = (1 - q)F(t')$, and the distribution of $X|X \geq \hat{x}^*$ are the same under both $\hat{H}_{G^{**}}$ and $H_{G^{**}}$. Therefore, $\mathbb{E}(\phi(X)|X \geq \hat{x}^*)$ and $\mathbb{E}(X|X < \hat{x}^*)$ are the same in both equilibria. The distribution of $X|X < \hat{x}^*$ in the equilibrium with $H_{G^{**}}$ is a mean-preserving spread of

the distribution of $X|X < \hat{x}^*$ in the equilibrium with $\hat{H}_{G^{**}}$, because the latter distribution is degenerate at $\Upsilon(t'; F)$. Since firm value is convex in the posterior mean, the ex ante expected firm value is greater under $H_{G^{**}}$ than under $\hat{H}_{G^{**}}$. ■

Lemma B.9 $\hat{x}^* = \mathbb{E}(V|t \leq V \leq a_2)$.

Proof of Lemma B.9. If $t = a_2$, then $t \leq \hat{x}^* \leq a_2$ implies $\hat{x}^* = t = a_2$ and the claim holds. So, for the remainder of the proof, suppose that $t < a_2$.

We can decompose the expectation $\int_{[0, \bar{v}]} x dG^*(x) = \int_{[0, \hat{x}^*]} x dG^*(x) + \int_{[\hat{x}^*, a_2]} x dG^*(x) + \int_{(a_2, \bar{v}]} x dG^*(x)$.

Let us study separately each of these three terms:

- $\int_{(a_2, \bar{v}]} x dG^*(x) = \bar{v} - a_2 G^*(a_2) - [S(\bar{v}; G^*) - S(a_2; G^*)] = \bar{v} - a_2 F(a_2) - [S(\bar{v}; F) - S(a_2; F)] = \int_{a_2}^{\bar{v}} x dF(x)$, where the first and third equalities follow from integration by parts, while the second equality from Lemma B.7 ($G^*(a_2) = F(a_2)$), from condition [MPS] ($S(\bar{v}; G^*) = S(\bar{v}; F)$), and from the definition of a_2 ($S(a_2; G^*) = S(a_2; F)$).
- $\int_{[0, \hat{x}^*]} x dG^*(x) = \hat{x}^* G^*(\hat{x}^*_-) - [S(t; G^*) + \int_t^{\hat{x}^*} G^*(x) dx] = \hat{x}^* F(t) - S(t; F) - (\hat{x}^* - t)F(t) = \int_0^t x dF(x)$, where the first and third equalities follow from integration by parts, while the second equality from Lemma B.8 ($G^*(x) = F(t)$ for all $x \in [t, \hat{x}^*]$) and the definition of t ($S(t; G^*) = S(t; F)$).
- $\int_{[\hat{x}^*, a_2]} x dG^*(x) = a_2 G^*(a_2) - \hat{x}^* G^*(\hat{x}^*_-) - \int_{\hat{x}^*}^{a_2} G^*(x) dx = a_2 F(a_2) - \hat{x}^* F(t) - (a_2 - \hat{x}^*)F(a_2) = \hat{x}^* [F(a_2) - F(t)]$, where the first equality follows from integration by parts, while the second equality from Lemma B.8 ($G^*(\hat{x}^*_-) = F(t)$) and Lemma B.7 ($G^*(x) = F(a_2)$ for all $x \in [\hat{x}^*, a_2]$).

Since G^* satisfies [MPS], $\int_{[0, \bar{v}]} x dG^*(x) = \int_0^{\bar{v}} x dF(x) = \mathbb{E}(V)$. Combining the three previous observations,

$$\int_0^t x dF(x) + \hat{x}^* [F(a_2) - F(t)] + \int_{a_2}^{\bar{v}} x dF(x) = \int_0^{\bar{v}} x dF(x),$$

which yields the desired result $\hat{x}^* = \int_t^{a_2} x dF(x) / [F(a_2) - F(t)] = \mathbb{E}(V|t \leq V \leq a_2)$. ■

Lemma B.10 $\phi(\hat{x}^*) - c = \phi(\mu(nd; G^*))$.

Proof of Lemma B.10. In the proof of Lemma B.9, we have seen that $\int_{[0, \hat{x}^*]} x dG^*(x) = \int_0^{\hat{x}^*} x dF(x)$.

By Lemma B.8, we know that $G^*(\hat{x}^*_-) = F(t)$. Combining these two facts, we conclude that $\mu(nd; G^*) =$

$\Upsilon(t; F)$. As explained at the beginning of this section, Lemmas B.6, B.7, and B.8 imply that the distribution of posterior expectations H_{G^*} is given by (B.14). Then, the integral of the c.d.f. is

$$S(x; H_{G^*}) = \begin{cases} 0 & \text{if } x \in [0, \Upsilon(t; F)) \\ q(x - \mathbb{E}(V)) + (1 - q)[S(t; F) + (x - t)F(t)] & \text{if } x \in [\Upsilon(t; F), \hat{x}^*) \\ q(x - \mathbb{E}(V)) + (1 - q)[S(t) + (\hat{x}^* - t)F(t) + (x - \hat{x}^*)F(a_2)] & \text{if } x \in [\hat{x}^*, a_2] \\ q(x - \mathbb{E}(V)) + (1 - q)S(x; F) & \text{if } x \in (a_2, \bar{v}] \end{cases} \quad (\text{B.22})$$

By way of contradiction, suppose that at the optimum the marginal discloser strictly prefers disclosure, $\phi(\hat{x}^*) - c > \phi(\Upsilon(t; F))$. We argue that if the inequality is strict, then ex ante firm value can be increased strictly by changing the measurement system. We consider separately the cases $t < a_2$ and $t = a_2$.

Case $t < a_2$. In this case, Lemma B.9 implies $t < \hat{x}^* < a_2$, because $\hat{x}^* = \mathbb{E}(V|t \leq V \leq a_2)$ and F admits a density that is strictly positive over the entire support of V . Therefore, we can keep t fixed while slightly decreasing a_2 (and decreasing $\hat{x}^* = \mathbb{E}(V|t \leq V \leq a_2)$ accordingly), in a way that preserves the inequality $\phi(\hat{x}^*) - c > \phi(\Upsilon(t; F))$. The expected ex ante firm value is

$$\begin{aligned} & \int_{\Upsilon(t; F)}^{\hat{x}^*} \phi''(x) \{q(x - \mathbb{E}(V)) + (1 - q)[S(t; F) + (x - t)F(t)]\} dx \\ & + \int_{\hat{x}^*}^{a_2} \phi''(x) \{q(x - \mathbb{E}(V)) + (1 - q)[S(a_2; F) - (a_2 - x)F(a_2)]\} dx \\ & + \int_{a_2}^{\bar{v}} \phi''(x) \{q(x - \mathbb{E}(V)) + (1 - q)S(x; F)\} dx, \end{aligned} \quad (\text{B.23})$$

where in the interval $[\hat{x}^*, a_2]$ we have substituted $S(t) + (\hat{x}^* - t)F(t) + (x - \hat{x}^*)F(a_2) = S(a_2; F) - (a_2 - x)F(a_2)$, which follows from $\hat{x}^* = \mathbb{E}(V|t \leq V \leq a_2)$. The derivative of (B.23) with respect to a_2 is

$$-(1 - q) \int_{\hat{x}^*}^{a_2} \phi''(x)(a_2 - x)f(a_2)dx < 0.$$

Since the derivative is negative, a decrease at the margin in a_2 would increase the ex ante firm value. Therefore, the original G^* is not optimal.

Case $t = a_2$. By Lemma B.9, in this case $t = \hat{x}^* = a_2$. By Lemma B.4, $t > 0$. So, we can keep $t = a_2 = \hat{x}^*$ and slightly decrease \hat{x}^* while preserving the inequality $\phi(\hat{x}^*) - c > \phi(\Upsilon(\hat{x}^*; F))$ (by

continuity). The expected ex ante firm value is

$$\begin{aligned} & \int_{\Upsilon(\hat{x}^*; F)}^{\hat{x}^*} \phi''(x) \{q(x - \mathbb{E}(V)) + (1 - q)[(x - \hat{x}^*)F(\hat{x}^*) + S(\hat{x}^*; F)]\} dx \\ & + \int_{\hat{x}^*}^{\bar{v}} \phi''(x) \{q(x - \mathbb{E}(V)) + (1 - q)S(x; F)\} dx. \end{aligned} \tag{B.24}$$

The derivative of (B.24) with respect to \hat{x}^* is

$$(1 - q) \int_{\Upsilon(\hat{x}^*; F)}^{\hat{x}^*} \phi''(x)(x - \hat{x}^*)f(\hat{x}^*)dx < 0,$$

where we have used the fact that $q(\Upsilon(\hat{x}^*; F) - \hat{x}^*) + (1 - q)[(\Upsilon(\hat{x}^*; F) - \hat{x}^*)F(\hat{x}^*) + S(\hat{x}^*)] = 0$ by the definition of $\Upsilon(\hat{x}^*; F)$. Since the derivative is negative, a decrease of \hat{x}^* at the margin increases ex ante firm value, so the original measurement G^* is not optimal. ■

Bibliography

Acharya, Viral V, Peter DeMarzo, and Ilan Kremer, “Endogenous information flows and the clustering of announcements,” *American Economic Review*, 2011, 101 (7), 2955–79.

Arya, Anil, Jonathan C. Glover, and K. Sivaramakrishnan, “Interaction between Decision and Control Problems and the Value of Information,” *The Accounting Review*, 1997, 72 (4), 561–574.

Aumann, Robert J and Michael Maschler, “Repeated games with incomplete information,” 1995.

Bertomeu, Jeremy and Edwige Cheynel, “Asset measurement in imperfect credit markets,” *Journal of Accounting Research*, 2015, 53 (5), 965–984.

— **and Iván Marinovic**, “A theory of hard and soft information,” *The Accounting Review*, 2015, 91 (1), 1–20.

— **, Igor Vaysman, and Wenjie Xue**, “Efficient Disclosure Laws,” 2016.

Beyer, Anne and Ilan Guttman, “Voluntary Disclosure, Manipulation, and Real Effects,” *Journal of Accounting Research*, 2012, 50 (5), 1141–1177.

Brandenburger, Adam and Ben Polak, “When managers cover their posteriors: Making the decisions the market wants to see,” *The RAND Journal of Economics*, 1996, pp. 523–541.

- Cooper, W, J Ho, J Hunter, and R Rodgers**, “The impact of the foreign corrupt practices act on internal control practices,” *Journal of Accounting, Auditing and Finance*, 1985, 9 (1), 22–40.
- Dechow, Patricia M, Weili Ge, Chad R Larson, and Richard G Sloan**, “Predicting material accounting misstatements,” *Contemporary accounting research*, 2011, 28 (1), 17–82.
- DeMarzo, Peter, Ilan Kremer, and Andrzej Skrzypacz**, “Test Design and Minimum Standards,” 2017. Working paper.
- Dworczak, Piotr and Giorgio Martini**, “The Simple Economics of Optimal Persuasion,” *Journal of Political Economy*, 2018, *forthcoming*.
- Dye, Ronald**, “Optimal Disclosure Decisions When There are Penalties for Nondisclosure,” 2018.
- Dye, Ronald A.**, “Disclosure of Nonproprietary Information,” *Journal of Accounting Research*, 1985, 23 (1), 123–145.
- Dye, Ronald A and John S Hughes**, “Equilibrium voluntary disclosures, asset pricing, and information transfers,” *Journal of Accounting and Economics*, 2018, 66 (1), 1–24.
- Einhorn, Eti**, “The nature of the interaction between mandatory and voluntary disclosures,” *Journal of Accounting Research*, 2005, 43 (4), 593–621.
- , “Competing Information Sources,” *The Accounting Review*, 2017.
- **and Amir Ziv**, “Intertemporal Dynamics of Corporate Voluntary Disclosures,” *Journal of Accounting Research*, 2008, 46 (3), 567–589.
- **and —**, “Biased Voluntary Disclosure,” *Review of Accounting Studies*, *forth.*, 2011.
- Files, Rebecca, Edward P Swanson, and Senyo Tse**, “Stealth disclosure of accounting restatements,” *The Accounting Review*, 2009, 84 (5), 1495–1520.
- Friedman, Henry L, John S Hughes, and Beatrice Michaeli**, “Optimal Reporting When Additional Information Might Arrive,” 2015. Working paper.
- Gao, Pingyang and Gaoqing Zhang**, “Accounting manipulation, peer pressure, and internal control,” *The Accounting Review*, 2018.
- **and Pierre Jinghong Liang**, “Informational feedback, adverse selection, and optimal disclosure policy,” *Journal of Accounting Research*, 2013, 51 (5), 1133–1158.

— and — , “Accounting Information, Renegotiation, and Debt Contracts,” 2018.

Gentzkow, Matthew and Emir Kamenica, “A Rothschild-Stiglitz Approach to Bayesian Persuasion,” *American Economic Review*, May 2016, 106 (5), 597–601.

Gigler, Frank, Chandra Kanodia, Haresh Sapra, and Raghu Venugopalan, “Accounting conservatism and the efficiency of debt contracts,” *Journal of accounting research*, 2009, 47 (3), 767–797.

— , — , — , and — , “How frequent financial reporting can cause managerial short-termism: An analysis of the costs and benefits of increasing reporting frequency,” *Journal of Accounting Research*, 2014, 52 (2), 357–387.

Glode, Vincent, Christian C Opp, and Xingtang Zhang, “Voluntary disclosure in bilateral transactions,” *Journal of Economic Theory*, 2018, 175, 652–688.

Goex, Robert F. and Alfred Wagenhofer, “Optimal impairment rules,” *Journal of Accounting and Economics*, 2009, 48 (1), 2–16.

Guttman, Ilan, Ilan Kremer, and Andrzej Skrzypacz, “Not only what but also when: A theory of dynamic voluntary disclosure,” *American Economic Review*, 2014, 104 (8), 2400–2420.

— , **Ohad Kadan, and Eugene Kandel**, “A Rational Expectations Theory of Kinks in Financial Reporting,” *The Accounting Review*, 2006, 81 (4), 811–848.

Hart, Sergiu, Ilan Kremer, and Motty Perry, “Evidence Games: Truth and Commitment,” *American Economic Review*, March 2017, 107 (3), 690–713.

Heinle, Mirko S and Robert E Verrecchia, “Bias and the Commitment to Disclosure,” *Management Science*, 2015, 62 (10), 2859–2870.

Heinle, Mirko Stanislav, Delphine Samuels, and Daniel J Taylor, “Proprietary Costs and Disclosure Substitution: Theory and Empirical Evidence,” 2018.

Holmström, Bengt and Jean Tirole, “Financial Intermediation, Loanable Funds, and Real Sector,” *Quarterly Journal of Economics*, 1997, 112, 663–692.

Huang, Zeqiong, “Optimal Reporting Systems with Investor Information Acquisition,” 2016.

Hummel, Patrick, John Morgan, and Phillip Stocken, “Optimal firm (non-) disclosure,” 2016.

Ijiri, Yuji, *Theory of accounting measurement* number 10, American Accounting Association, 1975.

- Jiang, Xu and Ming Yang**, “Properties of optimal accounting rules in a signaling game,” *Journal of Accounting and Economics*, 2017, 63 (2-3), 499–512.
- Jung, Woon-Oh and Young K. Kwon**, “Disclosure When the Market Is Unsure of Information Endowment of Managers,” *Journal of Accounting Research*, 1988, 26 (1), 146–153.
- Kamenica, Emir and Matthew Gentzkow**, “Bayesian Persuasion,” *American Economic Review*, 2011, 101 (6), 2590–2615.
- Kanodia, Chandra and Haresh Sapra**, “A real effects perspective to accounting measurement and disclosure: Implications and insights for future research,” *Journal of Accounting Research*, 2016, 54 (2), 623–676.
- Kanodia, Chandra S.**, “Effects of Shareholder Information on Corporate Decisions and Capital Market Equilibrium,” *Econometrica*, 1980, 48 (4), 923–953.
- , “Accounting Disclosure and Real Effects,” *Foundations and Trends in Accounting*, 2006, 1 (3), 167–258.
- , **Rajdeep Singh, and Andrew E Spero**, “Imprecision in accounting measurement: Can it be value enhancing?,” *Journal of Accounting Research*, 2005, 43 (3), 487–519.
- Laux, Volker**, “Effects of litigation risk on board oversight and CEO incentive pay,” *Management Science*, 2010, 56 (6), 938–948.
- **and Phillip C. Stocken**, “Managerial reporting, overoptimism, and litigation risk,” *Journal of Accounting and Economics*, 2012, 53 (3), 577 – 591.
- Li, Edward Xuejun**, “Revealing future prospects without forecasts: The case of accelerating material contract filings,” *The Accounting Review*, 2013, 88 (5), 1769–1804.
- Lu, Tong and Haresh Sapra**, “Auditor conservatism and investment efficiency,” *The Accounting Review*, 2009, 84 (6), 1933–1958.
- Marinovic, Ivan**, “Internal control system, earnings quality, and the dynamics of financial reporting,” *The RAND Journal of Economics*, 2013, 44 (1), 145–167.
- Michaeli, Beatrice**, “Divide and inform: Rationing information to facilitate persuasion,” *The Accounting Review*, 2017, 92 (5), 167–199.
- Quigley, Daniel and Ansgar Walther**, “Inside and Outside Information,” 2018. Working Paper.

Rappoport, Daniel, “Evidence and Skepticism in Verifiable Disclosure Games,” 2017. Working paper.

Verrecchia, Robert E., “Discretionary Disclosure,” *Journal of Accounting and Economics*, 1983, 5, 179–194.

— **and Joseph Weber**, “Redacted disclosure,” *Journal of Accounting Research*, 2006, 44 (4), 791–814.

Viscusi, W Kip, “A note on” lemons” markets with quality certification,” *The Bell Journal of Economics*, 1978, pp. 277–279.