Covenant Tightness and Conservatism*

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Abstract

We investigate the tightness of covenants in debt contracts and study how the informational properties of accounting reports such as the degree of accounting conservatism affects such covenants. We incorporate two decisions of the borrower after a debt contract is signed. One is asset substitution decision, which is unverifiable and value destroying; the other is an interim investment decision, which is verifiable and value-enhancing conditional on sufficiently favorable interim accounting reports.

We show that the covenant is optimally set "too tight" *ex-ante* in the sense that it forgoes some gains from the interim investment in order to combat asset substitution. The optimal level of the covenant increases as renegotiation cost decreases, as project risk increases, and as the propensity for asset substitution increases. Moreover, we find that conservatism is more likely to be value enhancing when the borrower can engage in unverifiable asset substitution.

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Covenants are initially set too tight and violations are often subsequently waived by the lender in lieu of concessions made by the borrower. Chen and Wei (1993) and Beneish and Press (1993) document that about 50% of firms that violate covenants get waivers from their lenders. Chava and Roberts (2008) document that in any given quarter, about 15% to 20% of loans violate a covenant. Dichev and Skinner (2002) find that 30% of firms in their sample violate their covenants while Nini et al (2007) find that 25% of public firms reported covenant violations between 1996 and 2005. They also report that it is the technical accounting based covenants that are the ones more frequently violated and waived. A priori, it is not clear why covenants should be set tight in the first place if violations are only to be waived in a subsequent renegotiation process that consumes valuable resources, when the covenant can be set appropriately at inception to minimize the waiver of violations and the possibility of renegotiation. We provide an explanation for this phenomenon.

We develop an analytical model in which after borrowing to finance a project, the borrower can make two modifications to the project: one of the modifications is verifiable and therefore contractible while the other modification is not verifiable. We model the contractible project modification as an interim investment that is conditionally value enhancing while the non-verifiable project modification is modeled as asset substitution that is unconditionally value destroying. In our model, an accounting report is informative about the future prospects of the verifiable interim investment and in conjunction with the covenant determines whether the borrower or the lender will obtain the control rights over the verifiable interim investment decision. We study how covenants written on the accounting report can be set optimally to regulate these two types of project modifications.

One contribution of our paper is that it identifies a tradeoff between the interim investment and asset substitution and thus the covenant's role of balancing this tradeoff. Specifically, in our model, the borrower's appetite for asset substitution does not directly depend on whether or not the interim investment is undertaken. Therefore, the two project modifications that we examine are not directly linked. Given this decoupling, the covenant that regulates the interim investment decision cannot possibly affect asset substitution directly. However, an indirect link exists between the contractible interim investment decision and the non-contractible asset substitution. This indirect link enables the covenant that directly regulates the interim investment to also indirectly influence the borrower's incentive to engage in asset substitution. The indirect link arises through the covenant's effect on the face value of debt (or equivalently the implicit interest rate). When the covenant is tighter, the likelihood of the interim investment being undertaken decreases. The lender therefore assesses a lower project risk and is willing to accept a lower face value of debt when the covenant is tighter. The lower face value of debt, in turn, decreases the borrower's incentive to engage in asset substitution. Thus, the covenant plays two roles – it directly regulates the interim investment decision that is contractible and indirectly disciplines unverifiable asset substitution. We find that unverifiable asset substitution causes optimal covenants to be tighter than when asset substitution is verifiable. A tighter covenant implies underinvestment in the sense that the interim investment is not undertaken in some circumstances even though the project's future prospects, as assessed from the accounting report, are favorable enough that it ought to be undertaken from a pure interim investment efficiency perspective. Thus, the optimal covenant tolerates underinvestment in the contractible risk arising from interim investment to alleviate value destruction from non-verifiable asset substitution risk. Additionally, we identify the determinants of covenant tightness. We find that the levels of covenants are higher for more risky projects, for lower debt renegotiation costs, or for larger asset substitution opportunities.

Another contribution of our paper is on the literature on conservatism. First, we find that debt contract efficiency considerations induce a demand for conservative accounting that is greater when asset substitution is unverifiable than when it is verifiable. Second, we find that even when asset substitution is verifiable and hence can be precluded, conservative accounting can still be value enhancing. We reconcile this result to the result in Gigler et al (2009) who show that in a world of symmetric information and full verifiability, debt contract efficiency considerations in the context of a liquidation decision call for an accounting system that is always liberal.

Our paper connects with several strands of the theoretical literature on debt contracting. Garleanu and Zwiebel (2009) also study the design of debt contracts and find that it is optimal to allocate control rights *ex-ante* to the lender when borrowers are better informed than lenders about their capacity to transfer wealth from lenders. Our model differs from that of Garleanu and Zwiebel (2009) in several salient ways. First, in their signaling model, asset substitution is modeled as an exogenous wealth transfer from the lender to the borrower that the borrower is better informed about than the lender at contract inception, whereas in our setting, asset substitution is an endogenous unverifiable choice made by the borrower after the debt contract is signed. Second, in their model the covenant is not written on any public accounting report but assigns the entire control right *ex-ante* to either the lender or the borrower. In contrast, in our model, the covenant is written on the accounting report and assignment of control rights to the lender or the borrower depends on whether or not the covenant is violated, thus permitting us to identify the determinants of covenant tightness. Finally, they do not study conservatism whereas our model identifies conditions under which conservative accounting enhances the efficiency of debt contracts. Callen et al (2016) also use a signaling model and find that when there is high information asymmetry at inception of contract about the borrower's ability to transfer wealth from lenders, conservatism and covenant tightness serve as signals for "Good" borrowers. These signaling models apply to settings with high information asymmetry between the parties at inception of a project, whereas our model is a moral hazard contracting model that is more relevant to settings where the parties are more symmetrically informed at inception of the project. This difference in settings and between the two types of models leads to different empirical predictions. In a signaling model, "Good" borrowers signal their type by offering tighter covenants than "Bad" borrowers, whereas our model predicts that covenant levels will increase for more risky projects and when renegotiation costs are lower.[1]

In other prior theoretical work on debt contracts, Gorton and Kahn (2000) examine asset substitution that occurs after an accounting report has been released, while our paper studies asset substitution that occurs prior to the release of the accounting report. Sridhar and Magee (1997) is an early paper that shows when the parties can observe a non-verifiable signal that is informative about future prospects, lenders anticipate opportunistic behavior by the borrower and demand stringent covenants. Their focus is on the impact of the informativeness of the accounting report and of managerial reporting discretion on the design of debt contracts.

There is also a recent literature that has examined the desirability of conservatism in a debt contracting context. Gigler et al (2009) examines the role of conservatism on the efficiency of debt contracts in a world of symmetric information and where the borrower cannot make project modifications. In this setting and in the context of a liquidation decision they find conservatism detracts from the efficiency of debt contracts. Li (2013) also examines a liquidation setting with a focus on the cost of renegotiation and finds that conservatism hurts debt contract efficiency when renegotiation costs are high. Jiang (2017) identifies regions where conservatism is beneficial by examining the informational properties of non-accounting information. Goex and Wagenhofer (2009) and Goex and Wagenhofer (2010) identify the determinants of optimal impairment rules, precision and bias used by the accounting system when a firm raises debt by pledging existing assets as collateral. Beyer (2012) relates aggregation to conservatism and identifies conditions that impact the desirability of conservative accounting. Unlike our model, these papers do not focus on tightness of covenants or on the impact of unverifiable asset substitution. Caskey and Hughes (2012) assess the impact of alternative fair value measures on covenant effectiveness in a project abandonment context, when the borrower can make a post-borrowing non-

[1] In these signaling models, a "Good" borrower is one with less ability to transfer wealth from lenders.

contractible project choice. In their model, the borrower's non-contractible project choice decision is directly impacted by whether the project is later continued or abandoned. In contrast, in our model the contractible interim investment decision has no direct impact on the borrower's non-contractible asset substitution incentives.

Section 2 introduces the model and characterizes the optimal benchmark covenant when asset substitution can be precluded. Section 3 allows the borrower to engage in asset substitution and characterizes the optimal covenant. Section 4 examines the role of conservatism. Section 5 concludes. The Appendix contains the proofs.

2. The Model

Consider a firm with exclusive rights to a project that needs investment of K at an initial date 0. The cash flow $\widetilde{\boldsymbol{X}}$ from the project is uncertain and realized at some terminal date 2. We assume that the entire investment for the project has to be obtained via debt raised at date 0 in the form of a zero coupon bond with face value D, to be repaid with interest at date 2 when the cash flow from the project is realized. The firm can borrow from a competitive debt market where the risk free interest rate is r. We assume that lenders as well as the residual claimants (henceforth, borrower) are risk-neutral so that lenders will lend to the firm if their expected repayment is at least $K(1+r)$. The implicit rate of interest at which lenders are willing to lend to the firm is then $\frac{D}{K}-1$, the endogenous face value of debt D divided by the initial investment of K, minus one.

There are two mutually exclusive and exhaustive states of the world – safe (S) and risky (R). The probability of the safe state is P_S and that of the risky state is $1-P_S$. At an interim date 1, the accounting system generates a report Z that is informative about whether the state of the world is S or R. The accounting report can take on a continuum of values on a support $[\underline{Z},\overline{Z}]$ and conditional on states S and R, it has conditional probability density functions, $f(Z|S;c)$ and $f(Z|R;c)$, respectively, and corresponding cumulative distribution functions $F(Z|S;c)$ and $F(Z|R;c)$, respectively. The unconditional density function and cumulative distribution function are denoted by $f(Z;c)$ and $F(Z;c)$, respectively. The parameter c captures the degree of conservatism of the reporting system. We will elaborate on the role of this conservatism parameter in Section 4 when we examine the role of conservatism. We assume that the accounting report satisfies the monotone likelihood ratio property (MLRP).

Assumption 1: The likelihood ratio $\frac{f(Z|S;c)}{f(Z|R;c)}$ is increasing in Z for all values of Z and c.

Assumption 1 guarantees that higher values of the accounting report increase the posterior assessment of the likelihood of the safe state S so that higher values of the accounting report constitute good news.

At the interim date 1, based on the information conveyed by the accounting report, the firm has the opportunity to make a project modification by undertaking an interim investment. For simplicity, we assume that the interim investment does not require any cash outlay. The interim investment decision that we want to study is one that is verifiable. Therefore, we let the interim investment shift the support of the distribution of cash flow $\widetilde{\boldsymbol{X}}$ from the project. When the state is safe, and the interim investment is not undertaken, the cash flow from the project is a positive constant X_S. If the interim investment is undertaken, the cash flow in the safe state increases to $X_S + \beta$ where β is a positive parameter. Conditional on the state being risky (R), the state of the world can be either bad (B) or not bad (NB), with conditional probabilities $Prob(B|R) = P_B$ and $Prob(NB|R) = 1 - P_B$. When the state is bad, and the interim investment is not undertaken, the cash flow is a positive constant X_B. If the interim investment is made, the cash flow in the bad state decreases by a positive parameter *T* to $X_B - T > 0$. Thus, the interim investment enhances the project's cash flow by β in the safe state but decreases the project's cash flow by *T* in the bad state. The interim investment is worth undertaking at date 1 when the probability of the safe state, as assessed from the accounting report, is sufficiently high. As the cash flow from the project is verifiable and changes with the interim investment, the interim investment decision also becomes verifiable.

In addition to the verifiable interim investment project modification, the model captures a second post-borrowing project modification in the form of asset substitution that is chosen by the borrower. We assume that asset substitution cannot be verified by a third party and hence is not contractible.[2] Asset substitution would be verifiable if it changes the cash flow from the project. We therefore do not allow asset substitution to shift the support of the distribution of cash flow $\widetilde{\boldsymbol{X}}$ from the project and instead model asset substitution as changing the probabilities of the project's cash flow outcomes. From Rothschild and Stiglitz (1970), such an unverifiable increase in project risk would entail moving probability mass from a Medium cash flow outcome to Low and High cash flow outcomes. Therefore, conditional on the state being not bad, we introduce three new states High (H), Medium (M) and Low (L) with project

[2] It does not matter whether the lender can observe the borrower's asset substitution choice or not. What is crucial is that asset substitution be unverifiable and hence not contractible.

cash flows X_H, X_M and X_L, respectively and $X_H > X_M > X_L$. Absent asset substitution, the conditional probabilities of the project cash flow in these three new states are:

$$Prob(X_H|NB) = \frac{q}{2};$$

$$Prob(X_M|NB) = 1 - q;$$

$$Prob(X_L|NB) = \frac{q}{2}.$$

Asset substitution, represented by the continuous variable a, increases project risk by moving probability mass conditional on state NB, in the sense described in Rothschild and Stiglitz (1970), from the Medium cash flow outcome (X_M) to the High (X_H) and Low (X_L) cash flow outcomes, by changing their probabilities as described below:

$$Prob(X_H|NB) = \frac{q}{2} + \gamma a;$$

$$Prob(X_M|NB) = 1 - q - a - \frac{k}{2}a^2;$$

$$Prob(X_L|NB) = \frac{q}{2} + (1-\gamma)a + \frac{k}{2}a^2.$$

The positive parameter γ captures the ease of asset substitution because a higher value of γ implies that more probability mass is shifted to the High state. The positive parameter k captures the cost of asset substitution because a higher value of k implies that more probability mass is moved to the Low state. The probability mass, conditional on state NB, moved by asset substitution to the High cash flow outcome state is γa while the probability mass (again, conditional on state NB) moved to the Low cash flow outcome is $(1-\gamma)a + \frac{k}{2}a^2$. As asset substitution increases, the probability mass moved to the Low cash flow outcome increases at an increasing rate.

We make the following assumptions on the parameter values.

Assumption 2: $\gamma < \frac{X_M - X_L}{X_H - X_L}; \frac{q}{2} \leq 1 - \gamma;\ \frac{k}{2} + \frac{q}{2} \leq \gamma; and\ all\ parameters\ are\ positive.$

The first inequality in Assumption 2 guarantees that asset substitution is value-destroying. The other inequalities in Assumption 2 ensure that the probabilities of the three cash flow outcomes (High, Medium and Low), conditional on state NB, are positive and less than one.

We have five possible cash flow outcomes X_S, X_H, X_M, X_L and X_B. The interim investment augments the cash flow in the Safe state by a positive constant β, and decreases the cash flow in the Bad state by a positive constant T such that $X_B - T > 0$.

Figure 1 describes the cash flow outcomes and the associated probabilities for the baseline project while Figures 2 through 4 describe how the distribution of cash flows from the project is affected by asset substitution and by the interim investment.

{Insert Figures 1 to 4 here.}

Given the above discussion, the probabilities of all the possible cash flow outcomes are summarized in Table 1.

{Insert Table 1 here.}

The cash flow outcomes are ordered as described below:

$$X_H > X_S \geq X_M > X_L > X_B.$$

To ensure that Assumption 1 on MLRP holds, that is, the Safe state is better than the Risky state, we further assume that

$$X_S > P_B X_B + (1 - P_B)\left[\frac{q}{2}(X_H + X_L) + (1 - q)X_M\right].$$

Asset substitution is unverifiable because it changes the probabilities of the three cash flow outcomes, X_H, X_M and X_L, without affecting the magnitude of any of the cash flow outcomes. In contrast, as the interim investment decision is verifiable, it changes the cash flow from the project, conditional on states Safe (S) and Bad (B).

The debt contract signed at date 0 is the triplet $\{K, D, Y\}$, where K is the amount borrowed at date 0, D is the face value of debt to be paid to the lender at date 2, and Y is the covenant. Violation of the covenant by the accounting report at date 1 transfers control of the interim investment decision to the lender. Else, the decision right vests with the borrower. At the interim date 1, after the accounting report is made public, the lender and borrower may renegotiate the terms of the initial debt contract. However, such renegotiation is costly as we describe later. If renegotiation occurs, the party that has the control rights to the interim investment decision may agree to waive its right in return for a change in the face value of debt. Let D^N denote the renegotiated face value of the debt to be repaid at Date 2. If no renegotiation occurs, then $D^N = D$. Finally, if the cash flow $\tilde{\boldsymbol{X}}$ realized at Date 2 exceeds D^N then full repayment of the face value of debt D^N occurs and the borrower gets the excess. Else, the borrower gets nothing and the lender takes the entire cash flow. We summarize the above discussion in the timeline below:

<u>Date 0</u>

- Lender and borrower sign a debt contract $\{K, D, Y\}$ that specifies the amount K borrowed, the face value of debt D and the covenant Y.
- Borrower engages in unverifiable asset substitution.

<u>Date 1</u>

- The accounting system releases a report Z that is informative but about whether the state of the world is Safe (S) or Risky (R).
- If the accounting report satisfies the covenant $(Z \geq Y)$, the borrower has the right to make the interim investment. Otherwise, the lender has control rights over the interim investment decision.
- The lender and borrower may renegotiate the terms of the initial debt contract at a cost. If costly renegotiation occurs, the party that has the control rights to the interim investment decision may agree to waive its right in return for a change in the face value of debt from D to a new D^N. If no renegotiation occurs, then $D^N = D$.

<u>Date 2</u>

- The terminal cash flow $\tilde{\boldsymbol{X}}$ from the project is realized**.** If $\tilde{\boldsymbol{X}}$ exceeds D^N then full repayment of the face value of debt D^N occurs and the borrower gets the excess. Else, the borrower gets nothing and the lender takes the entire cash flow $\tilde{\boldsymbol{X}}$.

2.1 Borrower and Lender Preferences

Let $B(a, D, I)$ and $L(a, D, I)$ denote borrower and lender date 0 expected payoffs, respectively, conditional on the interim investment being undertaken at date 1. Similarly, let $B(a, D, NI)$ and

$L(a, D, NI)$ denote borrower and lender date 0 expected payoffs, respectively, conditional on no interim investment at date 1. If $X_B < X_L < D < X_M \leq X_S < X_H$, then, using the probabilities listed in Table 1 and in Figures 1 through 4, the date 0 expected payoffs of the borrower and lender are given by the following equations:[3]

The date 0 expected payoff of the borrower given interim investment (*I*):

$$B(a, D, I) = P_S(X_S + \beta - D) + (1 - P_S)(1 - P_B)\left[\frac{q}{2} + \gamma a\right][X_H - D]$$

$$+(1 - P_S)(1 - P_B)\left[1 - q - a - \frac{ka^2}{2}\right][X_M - D]; \qquad (1)$$

The date 0 expected payoff of the borrower given no interim investment (*NI*):

$$B(a, D, NI) = P_S(X_S - D) + (1 - P_S)(1 - P_B)\left[\frac{q}{2} + \gamma a\right][X_H - D]$$

$$+(1 - P_S)(1 - P_B)\left[1 - q - a - \frac{ka^2}{2}\right][X_M - D]; \qquad (2)$$

The date 0 expected payoff of the lender given interim investment (*I*):

$$L(a, D, I) = \left\{P_S + (1 - P_S)(1 - P_B)\left[1 - \frac{q}{2} - (1 - \gamma)a - \frac{ka^2}{2}\right]\right\}D$$

$$+(1 - P_S)(1 - P_B)\left[\frac{q}{2} + (1 - \gamma)a + \frac{ka^2}{2}\right]X_L + (1 - P_S)P_B[X_B - T]; \qquad (3)$$

The date 0 expected payoff of the borrower given no interim investment (*NI*):

$$L(a, D, NI) = \left\{P_S + (1 - P_S)(1 - P_B)\left[1 - \frac{q}{2} - (1 - \gamma)a - \frac{ka^2}{2}\right]\right\}D$$

$$+(1 - P_S)(1 - P_B)\left[\frac{q}{2} + (1 - \gamma)a + \frac{ka^2}{2}\right]X_L + (1 - P_S)P_B X_B. \qquad (4)$$

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[3] We will later characterize a condition solely in terms of the parameters of the model that guarantees that the endogenous face value of debt is in the range (X_L, X_M) across all the settings that we study in this paper.

From a comparison of equations (1) and (2), it is clear that the borrower prefers to make the interim investment because she may reap the upside potential without worrying about the downside risk from the interim investment. Specifically, the interim investment may enhance the cash flow in the Safe state by the amount β. The borrower does not care that the interim investment may decrease the cash flow in the Bad state by an amount T, because the face value of debt exceeds the cash flow in the Bad state so that the borrower's payoff in the Bad state is zero regardless of the interim investment decision.

Moreover, from a comparison of equations (3) and (4), it is clear that the lender prefers that the interim investment not be undertaken because she may suffer the downside risk without reaping the upside potential from the interim investment. Specifically, the cash flow in the Bad state may be decreased by the interim investment. The lender does not care that the interim investment may increase the cash flow in the safe state because the lender's payoff is capped by the face value of debt that is lower than the cash flow X_S in the Safe state.

This divergence in preference over the verifiable interim investment is one source of conflict of interest between the borrower and the lender in the model.

The sum of the borrower and lender date 0 expected payoff is the social value of the project. Let $V(a, I)$ denote the social value of the project when the interim investment is undertaken. Then from equations (1) and (3),

$$V(a, I) \equiv B(a, D, I) + L(a, D, I)$$

$$= P_S(X_S + \beta) + (1 - P_S)(1 - P_B)\left[\frac{q}{2} + \gamma a\right] X_H + (1 - P_S)(1 - P_B)\left[1 - q - a - \frac{ka^2}{2}\right] X_M$$

$$+ (1 - P_S)(1 - P_B)\left[\frac{q}{2} + (1 - \gamma)a + \frac{ka^2}{2}\right] X_L + (1 - P_S)P_B[X_B - T]. \qquad (5)$$

Similarly, let $V(a, NI)$ denote the social value of the project when the interim investment is not undertaken. Then from equations (2) and (4),

$$V(a, NI) \equiv B(a, D, NI) + L(a, D, NI)$$

$$= P_S X_S + (1 - P_S)(1 - P_B)\left[\frac{q}{2} + \gamma a\right] X_H + (1 - P_S)(1 - P_B)\left[1 - q - a - \frac{ka^2}{2}\right] X_M$$

$$+(1-P_S)(1-P_B)\left[\frac{q}{2}+(1-\gamma)a+\frac{ka^2}{2}\right]X_L+(1-P_S)P_BX_B. \quad (6)$$

From equations (1) and (2), the borrower's date 0 expected payoff is decreasing in the face value of debt, regardless of whether the interim investment is undertaken or not undertaken. Similarly, from equations (3) and (4), regardless of whether the interim investment is undertaken, the lender's date 0 expected payoff is increasing in the face value of debt. These observations merely imply that the borrower prefers a lower interest rate while the lender prefers a higher rate of interest. From equations (5) and (6), it can be seen that regardless of whether the interim investment is made, , the date 0 social value of the project is independent of the face value of debt D, which is merely a transfer from the borrower to the lender. However, as we show next, the magnitude of this transfer will turn out to play a key role by impacting the incentives for asset substitution by the borrower.

From equations (3) and (4), the lender's date 0 expected payoff is decreasing in asset substitution so that the lender is hurt by asset substitution. From equations (1) and (2), the borrower's date 0 marginal expected payoff from asset substitution is independent of the interim investment and takes the form

$$B_a(D)=(1-P_S)(1-P_B)\{\gamma[X_H-D]-(1+ka)[X_M-D]\}. \quad (7)$$

Asset substitution moves probability mass from the Medium state to the High and Low states. The borrower does not care about the cash flow in the Low state as it is lower than the face value of debt. Given that the cash flow X_H in the High state is greater than the cash flow X_M in the medium state, equation (7) implies that for low values of asset substitution, the borrower's marginal date 0 expected payoff from asset substitution is positive. This divergence in borrower and lender preferences over non-verifiable asset substitution is the second source of conflict of interest between them in the model.

We, therefore, have two sources of conflict between the borrower and lender. One source of conflict is over the desirability of the verifiable interim investment. The interim investment enhances project value when the posterior probability of the Safe state based on the accounting report is sufficiently high so that the interim investment is conditionally desirable from a social perspective. An optimally chosen covenant can regulate the interim investment decision because it is verifiable. The other source of conflict arises from non-verifiable asset substitution.

From equation (7), the borrower's incentive to engage in unverifiable asset substitution is independent of whether the interim investment is undertaken or not undertaken. This decoupling precludes the possibility that regulation of the interim investment by a covenant will also *directly* impact asset substitution. We now proceed to characterize the optimal debt contract in a full information world.

2.2 First Best Benchmark

In a first best world, asset substitution is contractible. The first best contract will preclude asset substitution because it is contractible and value destroying. It is convenient to characterize the date 0 expected payoffs of the borrower and lender, absent renegotiation. We will later show that the first best covenant is renegotiation proof. Absent renegotiation and with asset substitution precluded, given a covenant Y, the date 0 expected payoffs $B^{FB}(D,Y)$ and $L^{FB}(D,Y)$ of the borrower and lender respectively, depend only on the face value of debt and the covenant. At the interim date 1, if the accounting reports violates the covenant, absent renegotiation, the lender will have the control rights to the interim investment and not allow the interim investment to be undertaken. On the other hand, if the covenant is met, absent renegotiation, the borrower will have the control rights to the interim investment and the interim investment will be undertaken. Therefore, from a date 0 perspective, the probability of the interim investment being undertaken is determined by the covenant Y. From Table 1 and Figures 1 through 4, this probability of undertaking the interim investment is

$$P_S[1-F(Y|S)]+(1-P_S)[1-F(Y|R)]. \tag{8}$$

The probability of the joint event where the interim investment is undertaken and the Safe State is realized is

$$P_S[1-F(Y|S)]. \tag{9}$$

And finally, the probability of the joint event where the interim investment is undertaken and the Bad State is realized is

$$(1-P_S)[1-F(Y|R)]P_B. \tag{10}$$

Using equations (1) and (2) and the probabilities characterized in expressions (8) through (10), with asset substitution precluded, the borrower's date 0 expected payoff is:

$$B^{FB}(D,Y)=B^{FB}(D,NI)+P_S[1-F(Y|S)]\beta. \tag{11}$$

Turning to the lender, using equations (3) and (4) and the probabilities characterized in expressions (8) through (10), with asset substitution precluded, the lender's date 0 expected payoff is:

$$L^{FB}(D,Y) = L^{FB}(D,NI) - (1-P_S)[1-F(Y|R)]P_B T. \qquad (12)$$

From equations (5) and (6) and the probabilities characterized in expressions in (8) through (10) above, the date 0 social value of the project is independent of the face value of debt and takes the form:

$$V(Y) = V(NI) + P_S[1-F(Y|S)]\beta - (1-P_S)[1-F(Y|R)]P_B T. \qquad (13)$$

We seek to maximize the borrower's date 0 expected payoff subject to the lender's participation constraint:

$$L^{FB}(D,Y) \geq K(1+r).$$

The Lagrange objective function for the optimal first best debt contract is therefore

$$\operatorname*{Max}_{D,Y,\lambda} B^{FB}(D,Y) + \lambda[L^{FB}(D,Y) - K(1+r)].$$

In the objective function above, λ is the endogenous Lagrange multiplier associated with the lender's participation constraint. The first order conditions for the first best program above (after dropping arguments of functions in the interest of brevity) are

$$B_D^{FB} + \lambda L_D^{FB} = 0; \qquad (14)$$

$$B_Y^{FB} + \lambda L_Y^{FB} = 0; \qquad (15)$$

$$L^{FB}(D,Y) = K(1+r). \qquad (16)$$

The face value of debt is merely a transfer from the borrower to the lender. Therefore

$$B_D^{FB} = -L_D^{FB}. \qquad (17)$$

Substituting equation (17) into equation (14) yields the optimal value of the Lagrange multiplier to be 1, which implies that in the first best case, the objective function is maximized when it places equal weights on the expected payoffs of the borrower and the lender. Inserting the optimal value of the Lagrange multiplier into equation (15) yields

$$L_Y^{FB} = -B_Y^{FB} \Leftrightarrow V_Y^{FB} = 0. \qquad (18)$$

Equation (18) characterizes the optimal first best covenant Y^{FB} by equating to zero the sum of marginal payoffs of the lender and borrower from tightening the covenant. Adding equations (11) and (12) and taking the derivative with respect to the covenant yields the following specific form for equation (18):

$$V_Y(Y) = (1 - P_S)P_B T f(Y|R) - P_S \beta f(Y|S) = 0. \tag{19}$$

Tightening of the covenant reduces the probability that the interim investment is undertaken. This reduction in the probability of interim investment implies that the project is less likely to suffer the loss (T) in cash flow that interim investment produces in the Bad state. The first term $(1 - P_S)P_B T f(Y|R)$ in equation (19) captures this marginal benefit that accrues to the lender from tightening the covenant. The reduction in the probability of interim investment also implies that the project is less likely to yield the enhancement (β) in cash flow that interim investment produces in the Safe state. The second term $P_S \beta f(Y|S)$ in equation (19) captures this marginal cost from tightening the covenant that is borne by the borrower. The optimal first best covenant trades off the marginal benefit to the lender against the marginal cost to the borrower of tightening the covenant. Per equation (18), at the optimal first best covenant, the marginal benefit of tightening the covenant is equal to its marginal cost. Equation (19) can be recast in terms of the likelihood ratio to yield

$$\frac{f(Y|S)}{f(Y|R)} = \frac{(1 - P_S)P_B}{P_S}\frac{T}{\beta}. \tag{20}$$

We now proceed to show that the optimal covenant in equation (20) is renegotiation proof. Let $Prob(S|Z)$ denote the posterior probability of the Safe state, given the accounting report and let $Gain(Z)$ denote the date 1 expected gain from undertaking the interim investment. Then

$$Prob(S|Z) = \frac{P_S f(Z|S)}{P_S f(Z|S) + [1 - P_S] f(Z|R)} \tag{21}$$

and

$$Gain(Z) = \beta Prob(S|Z) - T P_B [1 - Prob(S|Z)]. \tag{22}$$

A covenant Y would be renegotiation-proof if the date 1 expected gain from undertaking the interim investment is zero when the accounting report exactly meets the covenant. Inserting equation (21) into equation (22) and setting $Gain(Y)$ to zero yields equation (20) that verifies that the optimal first best covenant is renegotiation-proof. The above discussion leads to Proposition 1 below.

Proposition 1: *When asset substitution is verifiable and hence can be precluded, the optimal debt contract is such that*

- *(i)* *the efficient interim investment decision rule equates the expected marginal payoffs of the borrower and the lender from tightening the covenant;*
- *(ii)* *the covenant Y^{FB} is renegotiation-proof so that control rights to the interim investment decision are with the borrower when it is efficient to undertake the interim investment and control rights are with the lender when it is not efficient to undertake the interim investment; and*
- *(iii)* *the efficient covenant Y^{FB} is given by*

$$\frac{f\left(Y^{FB}\middle|S\right)}{f\left(Y^{FB}\middle|R\right)} = \frac{(1-P_S)P_B}{P_S}\frac{T}{\beta}.$$

3. Optimal Covenant with Unverifiable Asset Substitution

The debt contract signed at date 0 is the triplet $\{K, D, Y\}$ where ***K*** is the amount borrowed at Date 0, ***D*** is the face value of debt to be paid to the lender at Date 2 and ***Y*** is the covenant. Violation of the covenant by the accounting report at date 1 transfers rights to the interim investment decision to the lender. Else, the decision right vests with the borrower. It is convenient to first analyze the renegotiation game between the parties at date 1. Renegotiation will occur whenever the covenant is violated, but the accounting report is still favorable enough that the expected social gain of undertaking the interim investment exceeds the cost of renegotiation. In principle, renegotiation can also occur when the covenant is met and yet the accounting report is bad enough that the expected social gain from foregoing the interim investment exceeds the cost of renegotiation. We conjecture that renegotiation will occur only when the covenant is violated and not when it is met.[4]

In practice, renegotiation between lenders and borrowers is not costly. Such renegotiation costs could arise from costs of coordination among lenders or from resources expended by the lender to observe the extent of asset substitution. Let C^R denote the cost of renegotiation. We assume that $C^R < \beta$. Otherwise, no renegotiation would occur because the maximum potential gain from undertaking the interim investment is β. Let $Z(C^R)$ denote the solution to

[4] We later verify this conjecture.

$$Gain(Z) = C^R \tag{23}$$

Equation (23) implies that when the accounting report is higher than $Z(C^R)$ and yet fails to meet the covenant, the date 1 expected gain from undertaking the interim investment exceeds the cost of renegotiation so that the two parties can gain from renegotiation.

3.1 Analysis of Renegotiation

Given the initial debt contract, the accounting report Z and the chosen level of asset substitution, let the date 1 expected payoff of the lender be denoted by $L(a, D, Z, NI)$ when the interim investment is foregone, and by $L(a, D, Z, I)$ when the interim investment is undertaken, respectively. When the accounting report is in the renegotiation region $[Z(C^R), Y]$, the lender has control rights and prefers to not undertake the interim investment.. We assume, without of loss of generality, that the entire bargaining power at the interim date vests with the borrower. This assumption means that the borrower can make a take it or leave it offer to the lender. The lender will accept such offer as long as his date 1 expected payoff, given the accounting report, from accepting the offer is at least as great as the expected payoff from rejection or status quo. Therefore, the borrower will make an offer that keeps the lender indifferent between acceptance and rejection of the offer. The offer would have to be such that the lender is persuaded to waive control rights and permit the interim investment, i.e., the new face value of debt D^N that the borrower offers would have to meet the following indifference condition:

$$L(a, D, Z, NI) = L(a, D^N, Z, I) \tag{24}$$

Equation (24) implies that from a date 1 perspective, the lender is guaranteed to receive $L(a, D, Z, NI)$ for all accounting reports that violate the covenant and $L(a, D, Z, I)$ for all accounting reports that meet the covenant. Therefore, given a debt contract and a fixed level of asset substitution, the ex-ante date 0 expected payoff of the lender is the same as it would be without renegotiation. The lender is hurt by asset substitution and assesses a greater likelihood of suffering a loss from undertaking the interim investment when the accounting report is less favorable. Therefore, it is intuitive and reasonable to expect that the renegotiated face value of debt D^N will be such that it is higher when asset substitution is greater and when the covenant violation is more severe. This intuition is captured in Proposition 2 below.

Proposition 2

(i) *Renegotiation leads to a new face value of debt such that the lender is indifferent between undertaking the interim investment with the renegotiated face value of debt and foregoing the interim investment with the initially contracted face value of debt.*

(ii) *The renegotiated face value of debt is higher when the covenant violation is more severe and when asset substitution is greater.*

We now characterize the date 1 expected payoff of the borrower. Given the initial debt contract, the accounting report and the already chosen level of asset substitution, let the date 1 expected payoff of the borrower, absent renegotiation, be denoted by $B(a, D, Z, NI)$ when the interim investment is not undertaken, and by $B(a, D, Z, I)$ when the interim investment is undertaken, respectively. We make the following observations regarding the borrower's date 1 expected payoff when the accounting report is such that the initial contract is not renegotiated:

Observation 1: The borrower's date 1 expected payoff is $B(a, D, Z, NI)$ for all accounting reports in the region $Z \leq Z(C^R)$ where the covenant is violated, but the accounting report is not favorable enough to warrant renegotiation.

Observation 2: The borrower's date 1 expected payoff is $B(a, D, Z, I)$ for all accounting report in the region $Z > Y$ where the covenant is met.

In the region of renegotiation $[Z(C^R), Y]$, the borrower's date 1 expected payoff is $B(a, D^N, Z, I) - C^R$. Note that , the expected gain, net of renegotiation costs, from undertaking the interim investment in this region of renegotiation must be such that it is equal to the sum of the increase in borrower and lender date 1 expected payoffs from undertaking the interim investment. This equality is captured in equation (25) below:

$$B(a, D^N, Z, I) + L(a, D^N, Z, I) - [B(a, D, Z, NI) + L(a, D, Z, NI)] = Gain(Z) - C^R. \quad (25)$$

From equations (24) and (25), the borrower's date 1 expected payoff in the region of renegotiation $[Z(C^R), Y]$ is therefore

$$B(a, D^N, Z, I) = B(a, D, Z, NI) + Gain(Z) - C^R. \quad (26)$$

We can now characterize the borrower's date 0 expected payoff. Let $B^R(a, D, Y)$ denote the borrower's date 0 expected payoff that incorporates anticipated future gains from renegotiation and let $B^{NR}(a, D, Y)$

denote the borrower's date 0 expected payoff, absent any renegotiation and let the date 0 net expected gain from renegotiation be denoted by

$$\varphi(Y) \equiv \int_{Z(C^R)}^{Y} \{Gain(Z) - C^R\} f(Z) dZ. \qquad (27)$$

Then from Observations 1 and 2, and equations (25) and (26),

$$B^R(a, D, Y) = B^{NR}(a, D, Y) + \varphi(Y). \qquad (28)$$

Equation (28) implies that, given a debt contract, the borrower's date 0 expected payoff under renegotiation is the same as the borrower's Date 0 expected payoff absent renegotiation, augmented by the date 0 net expected gain $\varphi(Y)$ from renegotiation. From equation (27), this date 0 net expected gain from renegotiation is independent of asset substitution and the face value of debt. Further, the date 0 net expected gain from renegotiation increases as the covenant is increased, which makes covenant violation more likely and expands the range of accounting reports for which renegotiation occurs.

3.2 Asset Substitution Incentives

We want to characterize the asset substitution choice of the borrower and examine how it varies with elements of the debt contract. Using equations (1) and (2) and the probabilities characterized in expressions in (8) through (10), the date 0 expected payoff of the borrower is

$$B^R(a, D, Y) = B(a, D, NI) + \beta P_S[1 - F(Y|S)] + \varphi(Y). \qquad (28)$$

From equations (7), (27) and (28), the derivative of the borrower's date 0 expected payoff with respect to asset substitution is

$$B_a^R(a, D, Y) = B_a^{NR}(a, D, NI) = (1 - P_S)(1 - P_B)\{\gamma[X_H - D] - [1 + ka][X_M - D]\}. \qquad (29)$$

From equation (29) it is clear that the first order condition of the borrower's asset substitution choice problem is linear in asset substitution and yields a unique solution:

$$a(D) = \frac{1}{k}\left[\frac{\gamma(X_H - D)}{X_M - D} - 1\right]. \qquad (30)$$

It is clear from equation (30) that the asset substitution choice of the borrower is independent of the covenant and increases with the face value of debt D which leads us to Lemma 1.

Lemma 1

Absent renegotiation, given a debt contract $\{K, D, Y\}$:

(i) the borrower's asset substitution choice is $a(D) = \frac{1}{k}\left[\frac{\gamma(X_H - D)}{X_M - D} - 1\right]$*, and*

(ii) asset substitution incentives increase with the face value of debt: $\frac{\partial a(D)}{\partial D} > 0$.

Lemma 1 implies that the borrower's incentive to engage in asset substitution increases with the face value of debt - this result is consistent with the result in Green and Talmor (1986) who show that asset substitution increases with leverage. One implication of Lemma 1 is that in our model, the asset substitution choice of the borrower is not impacted directly by the covenant because the verifiable interim investment decision is decoupled from the unverifiable asset substitution choice of the borrower. However, we will establish that the covenant does have an indirect effect on asset substitution. This indirect effect will arise from a lower face value of debt that the lender would be prepared to accept when the covenant is tighter. A second implication of Lemma 1 is that in our model, given the initial debt contract, the lender can rationally anticipate the borrower's asset substitution choice that does not depend on the lender's conjecture. Therefore, the analysis is not affected by whether the lender can observe the borrower's asset substitution choice.

3.3 Determinants of Covenant Tightness

We derive the optimal debt contract by maximizing the date 0 expected payoff of the borrower, subject to the lender's participation constraint and the incentive compatibility constraint of the borrower that ensures that the borrower chooses asset substitution to maximize his expected payoff, given a debt contract. We have already established from equation (24) that the date 0 expected payoff of the lender does not depend on the prospect of renegotiation. We insert the incentive compatibility constraint into the objective function and into the lender's participation constraint so that the optimal full commitment debt contract is the solution to: [5]

$$\underset{D,Y}{Max} \quad B^{NR}(a(D), D, Y) + \varphi(Y)$$

[5] The first order approach of replacing the IC by the first order condition is valid as the borrower's asset substitution choice problem has been shown to have a unique solution in Lemma 1.

subject to

$$L^R(a(D), D, Y) = L(a(D), D, NI) - (1 - P_S)P_B[1 - F(Y|R)]T = K(1 + r).$$

The Lagrange objective function is therefore

$$\max_{D,Y,\lambda} B^{NR}(a(D), D, Y) + \varphi(Y) + \lambda[L^R(a(D), D, Y) - K(1 + r)].$$

where λ is the Lagrange multiplier associated with the participation constraint and $\varphi(Y)$ is the date 0 net expected gain from renegotiation. The first order conditions are:

$$B_D^{NR}(a(D), D, Y) + \lambda[L_D^R(a(D), D, Y) + L_a^R(a_D(D), D, Y)] = 0; \quad (31)$$

$$B_Y^{NR}(a(D), D, Y) + \varphi_Y + \lambda L_Y^R(a(D), D, Y) = 0; \quad (32)$$

$$L^R(a(D), D, Y) = K(1 + r). \quad (33)$$

Equate the two expressions for the multiplier from equations (31) and (32) to get (we drop the function arguments for brevity):

$$\lambda = \frac{B_Y^{NR} + \varphi_Y}{-L_Y^R} = \frac{-B_D^{NR}}{L_a^R a_D + L_D^R}. \quad (34)$$

The first term in the denominator of the right hand side of the second equality in equation (34) is negative and the face value of debt is merely a transfer from borrower to lender i.e., $L_D^R = -B_D^{NR}$. Therefore, the Lagrange multiplier is greater than 1, which means that the objective function places a greater weight on the lender's expected payoff when the borrower can engage in asset substitution. It also means that debt has a social cost in the presence of asset substitution. Cross multiply the fractions in equation (34), divide both sides by B_D^{NR} and use $L_D^R = -B_D^{NR}$ to express equation (34) as

$$L_Y^R + L_a^R a_D \left[\frac{-B_Y^{NR} - \varphi_Y}{B_D^{NR}}\right] + \varphi_Y = -B_Y^{NR}. \quad (35)$$

The right hand side of equation (35) captures the marginal cost while the left hand side captures the marginal benefit from tightening the covenant. The marginal cost is borne by the borrower and represents the foregone potential gain from undertaking the interim investment due to a tighter covenant. Comparison of equation (35) with the corresponding equation (18) that characterized the first best covenant shows that this marginal cost is identical to that in the first best case.

The marginal benefit from tightening the covenant on the LHS of equation (35) has three terms. The first term arises because a tighter covenant decreases the probability of undertaking the interim investment and helps the lender avoid the potential loss suffered in the Bad state when the interim investment is undertaken. A comparison with equation (18) shows that this marginal benefit was also present in the first best case. The second and third terms on the LHS of equation (35) are new components of marginal benefit that are not present in equation (18). The second term on the LHS of equation (35) is reproduced below:

$$L_a^R a_D \left[\frac{-B_Y^{NR} - \varphi_Y}{B_D^{NR}} \right].$$

The second term above arises because tightening the covenant loosens the participation constraint and allows for a lower face value of debt, which in turn lowers the borrower's incentive for asset substitution. This second component captures the marginal increase in the lender's expected payoff from the lower asset substitution induced by a tighter covenant, by keeping the borrower's expected payoff constant along the borrower's indifference curve in the space of face value of debt and the covenant. This additional marginal benefit of tightening the covenant arises from disciplining unverifiable asset substitution and causes the optimal covenant to be tighter than in the first best case. The covenant has no direct effect on asset substitution in our model. The effect of a tighter covenant on asset substitution is entirely indirect. A higher covenant level persuades the lender to accept a lower face value of debt because it lowers the probability of undertaking the interim investment and the lender prefers that the interim investment be not undertaken. A lower face value of debt, in turn, dampens the borrower's incentive to engage in asset substitution. The third term on the left hand side of equation (35) is the marginal benefit φ_Y from expansion of the renegotiation region as the covenant is increased. It captures the net marginal gain from renegotiating and undertaking the interim investment. Proposition 3 shows that the optimal covenant trades off the cost of inefficient asset substitution against the cost of inefficient underinvestment and that this tradeoff causes underinvestment, i.e., the optimal covenant is stricter than the one that induces an efficient interim investment decision.

Proposition 3

When the borrower can engage in unverifiable asset substitution,

(i) the objective function places a greater weight on the lender's expected payoff, i.e., debt has a social cost.

(ii) the optimal covenant Y^R is tighter than the first best covenant i.e., $Y^R > Y^{FB}$;

(iii) renegotiation takes place in the region $[Z(C^R), Y^R]$;

The net gain from renegotiation is higher when the cost of renegotiation is lower. Therefore, the optimal covenant increases when the cost of renegotiation is lower. Public debt contracts are not renegotiated because the cost of coordination among multiple lenders is high. Our model therefore predicts that covenant levels are higher under private debt contracts than under public debt contracts.

The marginal benefit from tightening the covenant is higher when asset substitution is more likely to matter. Furthermore, asset substitution does not affect the project's cash flow in the safe state. Therefore, our model predicts that covenant levels will increase for projects that are ex-ante assessed to be riskier. The marginal benefit from tightening the covenant is also higher when it is easier for the borrower to engage in asset substitution. Our model therefore predicts that covenant levels will increase when the borrower can more easily engage in asset substitution. This discussion leads us to Proposition 4 that is also illustrated in Figures 5 through 8.

{Insert Figures 5 to 8 here.}

Proposition 4

When the borrower can engage in unverifiable asset substitution, the optimal covenant Y^R increases when renegotiation costs are lower, the project is riskier and asset substitution incentives are higher.

We now list an assumption that guarantees that the initially contracted face value of debt D lies in the range (X_L, X_M).

Assumption 3: $X_M > w_1$ and $X_L < \overline{X_L}$ where $\{w_1, w_2\}$ are the two solutions (with $w_1 < w_2$) to

$$w = \frac{K(1+r) - (1-P_S)P_B(X_B - [1 - F(Y^{FC}|R]T) - (1-P_S)(1-P_B)(\frac{q}{2} + \psi(w))X_L}{P_S + (1-P_S)(1-P_B)(1 - \frac{q}{2} - \psi(w))},$$

where $\psi(w)$ is defined by

$$\psi(w) \equiv \frac{1}{2k}\left[\frac{\gamma(X_H - w)}{X_M - w} - 1\right]\left[1 - 2\gamma + \frac{\gamma(X_H - w)}{X_M - w}\right]$$

and $\overline{X_L}$ is the value of X_L at which $w_1 = 0$.

The characterization of Assumption 3 is not simple because the participation constraint of the lender is not linear in the initially contracted face value of debt. However, the intuition lying behind the assumption is straightforward. For the initially contracted face value of debt to lie in the range (X_L, X_M) we require that X_M be sufficiently large and X_L be sufficiently small while remaining positive. We next examine the impact of conservative accounting on the efficiency of debt contracts.

4. Role of Conservative Accounting

Tight covenants combat unverifiable asset substitution by giving up some profitable interim investment opportunities when the covenant is violated. Another dial that can be turned to make it more likely for covenants to be violated is the degree of conservatism embedded in the accounting report. We now examine to how the covenant tightness and the overall efficiency of the debt contract are affected by the degree of conservatism of the accounting system.

4.1 Informational Properties of Conservatism

Recall that $f(Z|S,c)$ and $F(Z|S,c)$ denote the conditional density and conditional cumulative distribution functions respectively, of the accounting report Z, given the safe state (S), where c is a parameter that represents how conservative the accounting system is. The corresponding conditional density and conditional distribution functions, given the risky state (R), are $f(Z|R,c)$ and $F(Z|R,c)$, respectively. We now specify three conditions on the measurement and reporting process that ensure that as c increases, the distribution and information content of the accounting report change in a way that is consistent with the accounting system becoming more conservative.

Condition C1: $F_C(Z|S;c) > 0 \; and \; F_C(Z|R;c) > 0$ i.e. $F(Z|S,c)$ and $F(Z|R,c)$ are increasing in c for all $Z \; and \; c$.

Condition C2: For any given Z, $\frac{f(Z|S;c)}{f(Z|R;c)} \; is \; increasing \; in \; c$.

Condition C3: $F_C(Z|S;c) = F_C(Z|R;c) \quad for \; all \; Z \; and \; c$.

Condition C1 is the same as condition A2 of Gigler et al (2009) and ensures that as the degree of conservatism increases, the distribution of accounting reports shifts to the left, conditional on each state of the world. It is consistent with the notion that conservatism imparts a downward bias to accounting reports. Condition C2, which is the same as condition A3 of Gigler et al (2009), ensures that as the degree of conservatism increases, the assessed distribution of cash flows given a fixed accounting report, becomes more favorable. Condition C3 is the same as condition A4 of Gigler et al (2009). It ensures that c is an index of unconditional conservatism in the sense that the downward shift in the distribution of accounting reports arising from increases in conservatism that is captured by Condition C1, is independent of the events being measured and therefore independent of the future cash flow of the firm. We emphasize, as noted in Gigler et al (2009), that the notion of unconditional conservatism captured by Condition C3 is not informationally benign. Changes in unconditional conservatism cannot be unraveled and affect the information content of the accounting reports and thus leave open the possibility of impacting the efficiency of debt contracts.

Let the uninformative accounting report that maintains prior beliefs be denoted by $Z^0(c)$. Therefore

$$\frac{f(Z^0(C))\,|S;c)}{f(Z^0(C))\,|R;c)} = 1. \tag{36}$$

When the accounting report is $Z^0(c)$, the likelihood ratio in equation (36) is 1 so that the prior probabilities of the safe and risky states are also the posterior probabilities. Condition C3 is equivalent to

$$f_C(Z|S;c) = f_C(Z|R;c) \quad for\ almost\ all\ Z\ and\ c. \tag{37}$$

Conditions C2 and C3 can be simultaneously hold only when

$$f_C(Z|S;c) = f_C(Z|R;c) < 0 \qquad for\ all\ Z > Z^0(C) \tag{38}$$

and

$$f_C(Z|S;c) = f_C(Z|R;c) > 0 \qquad for\ all\ Z < Z^0(C). \tag{39}$$

The inequalities in (38) and (39) imply that when accounting is made more conservative, the probabilities of all reports above the uninformative report $Z^0(c)$ decrease, while the probabilities of all reports below the uninformative report increase. Having specified how conservatism affects the distribution and informational properties of the accounting system, we turn to the problem of debt contracting and analyze how changes in the degree of accounting conservatism affect the optimal covenant and the efficiency of debt contract.

4.2 Demand for Conservatism Absent Asset Substitution

We first examine the first best case, where asset substitution is verifiable and hence is entirely precluded, so that the only efficiency that matters is that of the interim investment decision. When asset substitution is precluded, the date 0 maximized value of the project $V(Y^{FB}(c); c)$ is the sum of the expected payoff of the borrower and the lender when the decision rule is that the interim investment is undertaken if and only if the accounting report meets the covenant $Y^{FB}(c)$. The maximized value of the project depends on the optimal covenant $Y^{FB}(c)$ and on the degree of conservatism. We want to examine how changes in the degree of conservatism impact the optimal covenant and the maximized value of the project.

From equation (20) that is reproduced below, the optimal covenant $Y^{FB}(c)$ is chosen to maximize the value of the project and depends on the degree of conservatism:

$$\frac{f(Y|S;c)}{f(Y|R;c)} = \frac{(1-P_S)P_B}{P_S}\frac{T}{\beta}.$$

From condition C2, the likelihood ratio in the above equation increases with conservatism. Therefore, to maintain equality, the optimal covenant would have to be loosened. This loosening of the covenant is consistent with the intuition that the optimal covenant that is stated in terms of accounting reports will adjust in response to changes in the degree of conservatism of the accounting system that generates the accounting reports.

How does conservatism affect the likelihood of covenant violation? The direct impact of conservatism is to make it more likely for any fixed covenant to be violated. However, the optimal covenant is not fixed. We have shown that it becomes looser with conservatism, which in turn makes covenant violation less likely. Conservatism has therefore two effects on the likelihood of covenant violation. The direct effect increases the likelihood of covenant violation while the indirect effect via loosening of the optimal covenant decreases the likelihood of covenant violation. The net impact of the two opposing effects on the likelihood of covenant violation is ambiguous. This ambiguity implies that the effect of conservatism on the probability of undertaking the interim investment is also ambiguous because the interim investment is undertaken only when the covenant is met. The lender is hurt by the interim investment and would be willing to accept a lower face value of debt (implicitly, the interest rate) if the probability of interim investment is lower. The ambiguous effect of conservatism on the probability of undertaking the interim investment therefore implies that the impact of conservatism on the face value of debt is also ambiguous. This ambiguity calls into question the assertion that more conservative accounting would persuade lenders to lower the rate of interest on loans.

Even though the impacts of conservatism on the endogenous probability of covenant violation and face value of debt are ambiguous, we can still examine the impact of conservatism on the overall efficiency of the debt contract because the envelope theorem allows us to ignore how the endogenous variables vary with conservatism and focus on the direct effect of conservatism on the maximized value of the project. We now turn to examine the impact of conservatism on the maximized value of the project. The derivative of the maximized value of the project with respect to the degree of conservatism is

$$-P_S \beta F_c(Y^{FB}|S) + (1-P_S)P_B T F_c(Y^{FB}|R).$$

The first term in the above expression represents the marginal increase in the cost of false alarms from conservatism. The cost of false alarm is the value lost when the interim investment is not undertaken when the state is S. This cost increases with conservatism as the direct effect of conservatism makes it less likely that the interim investment will get undertaken. The second term in the above expression represents the marginal decrease in the cost of undue optimism from conservatism. The cost of undue optimism is the value lost when the interim investment is undertaken when the state is B. This cost decreases with conservatism.

Using Conditions C1 and C3, the sign of the derivative above is the same as the sign of

$$-P_S \beta + (1-P_S)P_B T.$$

Therefore, conservatism increases the value of the objective function if and only if

$$(1-P_S)P_B T > P_S \beta. \tag{40}$$

The left hand side of the inequality in (40) represents the date 0 expected loss from undertaking the interim investment while the right hand side of the inequality represents the date 0 expected gain from undertaking the interim investment. The inequality in (40) is met when the ex-ante belief at the time the project is initiated is that the expected net gain from undertaking the interim investment is negative, so that it is optimal to undertake the interim investment only if the accounting report is favorable enough that beliefs are sufficiently upgraded by the accounting report. This implies that the likelihood ratio at the optimal covenant Y^{FB} must be greater than 1 or equivalently that $Y^{FB}(c) > Z^0(c)$. The farther the likelihood ratio for an accounting report is from 1, the greater is the information content of the report.

Condition C2 implies that any increase in conservatism causes likelihood ratios to increase and therefore improves the information content at report values where the likelihood ratio is greater than 1. Therefore, conservatism enhances the informativeness of the accounting report at the optimal covenant when the inequality in (40) is satisfied.

Note that, if the ex-ante belief is reversed such that the inequality in (40) is not met, then , the expected net gain from undertaking the interim investment is positive at the time of project inception so that $Y^{FB}(c) < Z^0(c)$. This, in turn, implies that it is optimal to forego the interim investment only if the accounting report is unfavorable enough that beliefs are sufficiently downgraded by the report, and the likelihood ratio at the optimal covenant is less than 1. In this situation, the informativeness of the likelihood ratio is enhanced when the ratio is decreased further away from 1. Condition C2 would then imply that it more liberal reporting that would decrease the likelihood ratio and thus improve the information content of the accounting report at the optimal covenant. This discussion leads to Proposition 5 and Figure 9.

{Insert Figure 9 here.}

Proposition 5

When asset substitution is verifiable, conservatism

(i) causes the optimal covenant to decrease, and

(ii) enhances the efficiency of debt contracting when the ex-ante belief is such that the interim investment is unlikely to be undertaken.

This result in Proposition 5 that in even in the *first best* case, conservative accounting may be optimal is in contrast to the central result in Gigler et al (2009) that liberal accounting enhances the efficiency of debt contracts in a world of symmetric information and full verifiability. The two results can be reconciled by focusing on ex-ante beliefs and noting that in Gigler et al (2009), the interim decision is a liquidation decision, whereas we study an interim investment decision. In the context of a liquidation decision, it is reasonable to assume that ex-ante beliefs are such that liquidation is not optimal so that a deterioration in initial beliefs is required for liquidation to be optimal, which in turn causes liberal accounting to be optimal. However, in the context of an interim investment decision, it is not necessary that ex-ante beliefs be such that interim investment is optimal from a date 0 perspective. When ex-ante beliefs are such that interim investment is optimal from a date 0 perspective, then liberal accounting is optimal. However, when the ex-ante beliefs are such that interim investment is not optimal from a date 0 perspective, then conservative accounting is optimal. We now turn to examining the demand for conservative accounting when asset substitution is unverifiable.

4.3 Demand for Conservatism with Unverifiable Asset Substitution

The maximized Lagrange objective function when the borrower can engage in unverifiable asset substitution is

$$B(a(D^R), D^R, Y^R; c) + \varphi(Y^R; c) + \lambda^R[L(a(D^R), D^R, Y^R; c) - K(1+r)].$$

By the envelope theorem, conservative accounting is optimal if and only if

$$B_c^{NR} + \varphi_c + \lambda^R L_c^R = \varphi_c - P_S F_c(Y^R|S)\beta + \lambda^R(1-P_S)F_c(Y^R|R)P_B T > 0.$$

Using Conditions C1 and C3, the above inequality can be simplified so that conservatism increases the value of the maximized objective function when

$$\varphi_c + \lambda^R(1-P_S)P_B T > P_S\beta. \tag{41}$$

We compare the inequality in (41) to that in (40) when asset substitution was verifiable. The right hand sides of the inequalities in (40) and (41) are identical and represent the expected cost of increase in false alarms when conservatism increases. The LHS of the inequality in (40) and the second term on the LHS of the inequality in (41) capture the expected value of reduction in undue optimism from conservatism. Recall that the Lagrange multiplier is greater than 1 when asset substitution is unverifiable. Therefore, the second term on the LHS of the inequality in (41) is greater than the LHS of (40) which means that the expected value of reduction in undue optimism from conservatism is higher when asset substitution is unverifiable. This higher value of reduction in undue optimism arises because the objective function places a greater weight on the lender's expected payoff when asset substitution is not verifiable.

The preceding discussion implies that a sufficient condition for a greater demand for conservatism when asset substitution is unverifiable is that the first term φ_c on the left hand side of the inequality in (41) be positive. This term represents the change in the date 0 net expected gain from renegotiation induced by conservatism. It turns out that the sign of φ_c is not unambiguously positive as the sign depends on where the range $[Z(C^R), Y^R]$ of renegotiation lies relative to the uninformative report $Z^0(c)$. However, suppose that conservatism is value enhancing when asset substitution is verifiable. This positive role of conservatism arises when the first best covenant is greater than the uninformative report, i.e., $Y^{FB}(c) > Z^0(c)$. Note that the lower bound $Z(C^R)$ of the region of renegotiation is strictly greater than $Y^{FB}(c)$ because the lower bound is increasing in the cost of renegotiation and attains the value $Y^{FB}(c)$ when the cost of renegotiation is zero. Therefore, if conservatism is value enhancing in the first best case, then

$$Z(C^R) > Y^{FB}(c) > Z^0(c). \tag{42}$$

The inequality in (42) implies that the debt contract will be renegotiated only when the accounting report is such that prior beliefs are upgraded. Further, the extent of upgradation in beliefs required for the contract to be renegotiated is more than that required to undertake the interim investment in the first best case. In other words, from a date 0 perspective, it is less likely that the accounting report will be such that the debt contract will be renegotiated and interim investment undertaken than in the first best case. The implication is therefore that if conservatism is value enhancing in the first best case, it will continue to be value enhancing when asset substitution is unverifiable. This discussion leads to Proposition 6.

Proposition 6

When asset substitution is unverifiable, the demand for conservative accounting is higher.

Our model does not yield a clear prediction on how the demand for conservatism changes with the cost of renegotiation. This ambiguity arises because whether the net expected date 0 gain from negotiation increases with conservatism or not, depends on the location of the range $[Z(C^R), Y^R]$ of renegotiation relative to the uninformative report $Z^0(c)$.

The optimal covenant adjusts and becomes more lenient with conservatism. However, like in the first best case, the effects of conservatism on the likelihood of covenant violation, on the probability of undertaking the interim investment and on the face value of debt continue to remain ambiguous when asset substitution is not verifiable.

6. Conclusion

We have developed an analytical model to examine how covenants are optimally set in the presence of agency conflicts between borrowers and lenders. In our model, the covenant mediates the conflict of interest between the borrower and the lender over an interim investment decision that the borrower makes after the release of an accounting report. We show that the optimal covenant balances the trade-off between making an efficient interim investment decision and mitigating inefficient asset substitution, and is set "too tight" *ex ante* in order to mitigate the incidence of asset substitution. A “too tight” covenant implies that even when the covenant is violated, the accounting report can be favorable enough that assessed future prospects are such it is efficient to make the interim investment from a sequentially rational perspective. The debt contract is therefore renegotiated when the accounting system produces accounting reports that violate the covenant but are still favorable enough to warrant that interim investment be undertaken. We show that the prospect of renegotiation causes covenants to be even tighter initially. Thus, we provide an explanation for the empirically documented phenomenon that covenants are set “too tight.”

Our model generates several empirical predictions that are supported by findings in the extant empirical literature. We also generate some new empirical predictions. Regarding covenant tightness, the model predicts that it is positively associated with the borrower’s business risk and with asset substitution opportunities while it is negatively associated with the cost of asset substitution and with renegotiation costs. While conservatism causes the optimal covenant to be adjusted and made more lenient, our model is agnostic about whether the net effect of conservatism and a looser covenant increases or decreases the likelihood of covenant violation and the endogenous interest rate in the debt contract.

Christensen, Nikolaev, and Wittenberg-Moerman (2016) discuss two streams of empirical literature on debt and accounting. One stream (Ahmed et al 2002) uses the extent of timely loss recognition to measure the contracting value of conservatism. In our model, the value of conservatism comes from the high information content that conservatism imparts to favorable accounting reports at the cost of low information content for unfavorable reports. When covenants are set tight, the covenant is likely to be met only when the accounting report is favorable, and therefore conservatism adds value by improving the efficiency of the verifiable project modification that the covenant regulates. Thus, our results support the line of reasoning in the first stream of empirical literature.

A second stream argues that conservative reporting facilitates the more timely transfer of control rights to lenders via covenants when borrowers perform poorly (e.g., Wittenberg-Moerman 2008 and Nikolaev 2010). Our model incorporates transfer of control rights to the lender when the covenant is violated. It finds that the value of a tighter covenant comes from the lower face value of debt that the lender is persuaded to agree to in anticipation of more frequent transfer of control rights. The lower face value of debt, in turn, decreases the borrower’s appetite for asset substitution. However, while more frequent transfer of control to the lender adds value, two factors give us pause from concluding that the value of conservatism comes from more frequent transfer of control rights to the lender. First, we find that control rights are waived by the lender during renegotiation. Therefore the efficiency of the interim investment decision is not affected by the transfer of control rights to the lender. Further, the optimal covenant adjusts and becomes looser when the accounting system is more conservative, making it unclear whether conservatism results in more frequent transfer of control rights to the lender.

A stream of research in empirical accounting, e.g. Beatty, Weber and Yu (2008) and Zhang (2008), takes the stance that compared to public bond holders, private lenders have a stronger demand for conservatism due to their ability to closely monitor borrowers, However, as we discuss above, our model is agnostic about differential demand for conservatism across private and public debt settings. This, in turn, implies that the documented regularity in this stream of empirical research may reflect the *average* demand for conservatism by public bondholders and private lenders, respectively. The finding in Beatty, Weber, and Yu (2008) that contract modifications (income escalators) are more likely when agency costs of debt are higher is consistent with our result that covenant levels will be higher when the borrower's propensity for asset substitution is higher. We also show that in the presence of agency conflicts, efficiency considerations induce a greater demand for conservatism. This result supports the prescription in Watts (2003) that conservatism protects the interests of lenders in the presence of agency conflicts.

Appendix

List of Some Useful Expressions

$$B_{aD}^{NR}(a,D,Y) = (1-P_S)(1-P_B)[1-\gamma+ka] > 0$$

$$B_{aa}^{NR}(a,D,Y) = -k(1-P_S)(1-P_B)[X_M-D] < 0$$

$$\frac{da(D)}{dD} \equiv a_D = \frac{-B_{aD}^{NR}}{B_{aa}^{NR}} = \frac{[1-\gamma+ka]}{k[X_M-D]} > 0$$

$$a_D = \frac{\gamma[X_H-X_M]}{k[X_M-D]^2} > 0$$

$$B_D^{NR}(a,D,Y) = -P_S - Prob(X_H;a) - Prob(X_M;a)$$

$$= -P_S - (1-P_S)(1-P_B)\left[1-\frac{q}{2}-(1-\gamma)a-\frac{ka^2}{2}\right] < 0$$

$$B_Y^{NR}(a,D,Y) = -\beta P_S f(Y|S) < 0$$

$$L^R(a,D,Y) = L(a,D,NI) - (1-P_S)P_B[1-F(Y|R)]T$$

$$L_a^R(a,D,Y) = L_a(a,D,NI) = -(1-P_S)(1-P_B)[(1-\gamma)+ka]D < 0$$

$$L_D^R(a,D,Y) = P_S + Prob(X_H;a) + Prob(X_M;a) = -B_D^{NR}(a,D,Y) > 0$$

$$L_Y^R(a,D,Y) = (1-P_S)P_B f(Y|R)T > 0$$

Proof of Proposition 1

We start by reproducing equation (20) from the body of the paper below.

$$\frac{f(Y|S)}{f(Y|R)} = \frac{(1-P_S)P_B}{P_S}\frac{T}{\beta}. \tag{20}$$

MLRP implies that the LHS of the equation (20) is increasing in Y. The RHS is a positive constant. So a unique covenant Y^{FB} exists.

Proof of Proposition 2

Part (i) of the Proposition is established by equation (24).

Substitute for the LHS of equation (24) from equation (4) after replacing the prior probability of the Safe state by the posterior probability $Prob(S|Z)$. Similarly, substitute for the RHS of equation (24) from equation (3) after replacing the prior probability of the Safe state by the posterior probability $Prob(S|Z)$ and by replacing the initially contracted face value of debt by D^N. Then cancel the common terms to get

$$\left\{Prob(S|Z) + \big(1 - Prob(S|Z)\big)(1-P_B)\left[1 - \frac{q}{2} - (1-\gamma)a - \frac{ka^2}{2}\right]\right\}\{D^N - D\}$$

$$= \big(1 - Prob(S|Z)\big)P_B T. \tag{45}$$

The LHS of equation (45) is decreasing in asset substitution while the RHS is independent of asset substitution. Therefore D^N is increasing in asset substitution. The sign of the derivative of the LHS of equation (45) with respect to $Prob(S|Z)$ is given by the sign of

$$1 - (1-P_B)\left[1 - \frac{q}{2} - (1-\gamma)a - \frac{ka^2}{2}\right]. \tag{46}$$

The expression in (46) is positive because $(1-P_B)\left[1 - \frac{q}{2} - (1-\gamma)a - \frac{ka^2}{2}\right]$ is a probability and hence lower than 1. Therefore the LHS of equation (45) is increasing in $Prob(S|Z)$. The RHS of equation (45) is decreasing in $Prob(S|Z)$. Therefore D^N is decreasing in $Prob(S|Z)$, which in turn increases as the accounting report becomes more favorable. So we have established that D^N is decreasing in the accounting report.

Proof of Lemma 1

Part (i) of the Lemma has been proved in the body of the paper by noting that equation (30) is linear in the borrower's asset substitution choice. We now prove part (ii) of the Lemma below.

Let $a(D)$ solve the first order condition to the borrower's asset substitution choice. So

$$B_a^{NR}(a, D, NI) = (1 - P_S)(1 - P_B)\{\gamma[X_H - D] - [1 + ka(D)][X_M - D]\} = 0 \; for \; all \; D.$$

From the level curve above,

$$\frac{da(D)}{dD} \equiv a_D = \frac{-B_{aD}^{NR}}{B_{aa}^{NR}} = \frac{-(1 - P_S)(1 - P_B)[1 - \gamma + ka(D)]}{-k(1 - P_S)(1 - P_B)[X_M - D]} = \frac{[1 - \gamma + ka(D)]}{k[X_M - D]} > 0.$$

Proof of Proposition 3

Establishment of a Unique Optimal Covenant

Divide both sides of equation (35) by $-B_Y^{NR} - \varphi_Y$ to get

$$\frac{L_Y^R}{-B_Y^{NR} - \varphi_Y} + \frac{L_a^R a_D}{B_D^{NR}} = 1. \tag{47}$$

Denote the LHS of equation (47) by the function $\xi(a(D), D, Y)$, the first term of the LHS by $G(Y)$ and the second term on the LHS by $H(a(D), D)$. We will first show that $\xi(a(D), D, Y)$ is increasing in D by examining each of its two terms separately.

<u>First Term</u>

The first term is

$$G(Y) \equiv \frac{L_Y^R}{-B_Y^{NR} - \varphi_Y}. \tag{48}$$

Differentiate equation (27) with respect to the debt covenant to get

$$\varphi_Y = \{Gain(Y) - C^R\}f(Y) > 0. \tag{49}$$

Substitute for $Gain(Y)$ from equations (21) and (22) into equation (49) and use

$$f(Y) = P_S f(Y|S) + (1 - P_S) f(Y|R) \tag{50}$$

to get

$$\varphi_Y = \beta P_S f(Y|S) - TP_B[1 - P_S] f(Y|R) - C^R[P_S f(Y|S) + [1 - P_S] f(Y|R)]. \tag{51}$$

Substitute for φ_Y from (51) into equation (48). Use $L_Y^R = (1 - P_S) P_B f(Y|R) T$ and

$B_Y^{NR} = -\beta P_S f(Y|S]$ write equation (48) as

$$G(Y) = \frac{(1 - P_S) P_B f(Y|R) T}{TP_B[1 - P_S] f(Y|R) + C^R[P_S f(Y|S) + [1 - P_S] f(Y|R)]}. \tag{52}$$

Divide Numerator and denominator of (52) by $f(Y|R)$ to get

$$G(Y) = \frac{(1 - P_S) P_B T}{TP_B[1 - P_S] + C^R\left[P_S \dfrac{f(Y|S)}{f(Y|R)} + [1 - P_S]\right]}. \tag{53}$$

Note that $G(Y)$ is independent of D and asset substitution and by MLRP, decreasing in Y.

<u>Second Term</u>

The second term is

$$H(a(D), D) \equiv \frac{L_a^R a_D}{B_D^{NR}} = \left\{\frac{-B_{aD}^{NR}}{B_{aa}^{NR}}\right\} \frac{L_a^R}{B_D}$$

$$= \left\{\frac{[1 - \gamma + ka]}{k[X_M - D]}\right\} \frac{(1 - P_S)(1 - P_B)[(1 - \gamma) + ka] D}{P_S + (1 - P_S)(1 - P_B)\left[1 - \frac{q}{2} - (1 - \gamma) a - \frac{ka^2}{2}\right]}.$$

The numerator of $H(a(D), D)$ is increasing in asset substitution while the denominator is decreasing in asset substitution. Therefore H_a is positive.

The numerator of $H(a(D), D)$ is increasing in D while the denominator is decreasing in D. Therefore H_D is positive. So we have shown the following:

$$\xi_a = H_a > 0,$$

$$\xi_D = H_D > 0,$$

so that the total derivative of $\xi(a(D), D, Y)$ with respect to D is positive i.e.

$$\xi_D + \xi_a a_D > 0,$$

which means that the LHS of equation (47) is increasing in D, which in turn implies that for every Y, there exists a unique D that solves equation (47). Let $D(Y)$ solve (47) so that

$$\xi\big(a\big(D(Y)\big), D(Y), Y\big) = 1 \;\; for\ all\ Y. \tag{54}$$

Differentiate equation (54) with respect to Y to get

$$\xi_a a_D D'(Y) + \xi_D D'(Y) + \xi_Y = 0 \quad for\ all\ Y.$$

Therefore

$$D'(Y) = \frac{-\xi_Y}{\xi_a a_D + \xi_D}. \tag{55}$$

The denominator of the expression for $D'(Y)$ in equation (55) is positive. Since $H(a(D), D)$ is independent of Y,

$$\xi_Y = G'(Y) < 0 \tag{56}$$

so that the $D(Y)$ that solves (47) is increasing in Y.

Let $D^{PC}(Y)$ solve the PC. The lender's Date 0 expected payoff increases with the face value of debt and with the tightness of the covenant. Therefore $D^{PC}(Y)$ is decreasing in the covenant. As the $D(Y)$ that solves (47) and $D^{PC}(Y)$ have opposite slopes, existence of a unique intersection point Y^R is established.

That $Y^R > Y^{FB}$ is established by comparing equations (18) and (35) and noting that the LHS of equation (35) is greater than the LHS of equation (18), which proves part (i) of the proposition.

From equation (23), a covenant violation is not renegotiated when the accounting report is in the region $[Y^{FB}, Z(C^R)]$ which proves part (ii) of the proposition.

Part (iii) of the proposition was established in the discussion in the body of the paper following equation (34).

Proof of Proposition 4

Comparative Statics with respect to Cost of Renegotiation

We note from equation (53) that $G(Y)$ is decreasing in C^R and that by MLRP $G(Y)$ is decreasing in Y. Therefore

$$\frac{\partial Y}{\partial C^R} = \frac{-G_{C^R}}{G_Y} < 0. \tag{57}$$

The inequality in (57) and the fact that $D^{PC}(Y)$ is decreasing in the covenant implies that the optimal covenant Y^R falls as the cost of renegotiation increases.

From equation (22) and (23), it is clear that $Z(C^R)$ increases with the cost of renegotiation. That Y^R falls as the cost of renegotiation increases and $Z(C^R)$ increases with the cost of renegotiation implies that the region of renegotiation $[Z(C^R), Y^R]$ shrinks as the cost of renegotiation increases.

Comparative Statics with respect to Probability of Risky State

Divide equation (53) by $[1 - P_S]$ to get

$$G(Y) = \frac{P_B T}{TP_B + C^R \left[\frac{P_S}{[1 - P_S]} \frac{f(Y|S)}{f(Y|R)} + 1\right]}. \tag{58}$$

We note from equation (58) that $G(Y)$ is decreasing in P_S and that by MLRP $G(Y)$ is decreasing in Y. Therefore

$$\frac{\partial Y}{\partial P_S} = \frac{-G_{P_S}}{G_Y} < 0. \tag{59}$$

The inequality in (59) and the fact that $D^{PC}(Y)$ is decreasing in P_S implies that the optimal covenant Y^R falls as the P_S increases as the probability of the risky state increases.

Comparative Statics with respect to Ease of Asset Substitution

$G(Y)$ is independent asset substitution and decreasing in the covenant. Further, $\xi_a = H_a > 0$ and from equation (30), asset substitution is increasing in the parameter γ. Therefore in equation (35)

$$\frac{\partial Y}{\partial \gamma} > 0. \tag{60}$$

The lender's Date 0 expected payoff is decreasing in asset substitution and therefore in the parameter γ. So $D^{PC}(Y)$ is increasing in the parameter γ. This fact and the inequality in (60) imply that the optimal covenant Y^R increases as γ increases.

Proof of Proposition 5

The proof is contained the body of the paper in Section 4.2.

Proof of Proposition 6

Suppose that conservatism is value enhancing in the first best case so that the inequality in (40) is met. We want to show that the inequality in (41) is also met. As $\lambda^R > 1$, it suffices to show that $\varphi_C > 0$ when the inequality in (40) is met. We now proceed to characterize φ_C. Differentiate equation (27) with respect to the degree of conservatism to get

$$\varphi_c = \int_{Z^R(C^R)}^{Y} \frac{\partial}{\partial c}\{Gain(Z) - C^R\} f(Z)dZ, \quad (61)$$

where

$$[Gain(Z) - C^R]f(Z) = \beta P_S f(Z|S) - TP_B[1-P_S]f(Z|R) - C^R[P_S f(Z|S) + [1-P_S]f(Z|R)]. \quad (62)$$

Differentiate equation (62) with respect to the degree of conservatism to get

$$\frac{\partial}{\partial c}\{Gain(Z) - C^R\} = \beta P_S f_c(Z|S) - TP_B[1-P_S]f_c(Z|R) - C^R[P_S f_c(Z|S) + [1-P_S]f_c(Z|R)]$$

$$= [\beta - C^R]P_S f_c(Z|S) - [1-P_S][TP_B + C^R]f_c(Z|R). \quad (63)$$

Use equation (37) to express equation (63) as

$$\frac{\partial}{\partial c}\{Gain(Z) - C^R\} = \{\beta P_S - [1-P_S]TP_B - C^R\}f_c(Z|S)$$

$$= \{\beta P_S - [1-P_S]TP_B - C^R\}f_c(Z|R). \quad (64)$$

Insert equation (64) into equation (61) to get

$$\varphi_c = \{\beta P_S - [1-P_S]TP_B - C^R\}\{F_c(Y^R|S) - F_c(Z(C^R)|S)\}. \quad (65)$$

From inequality (39) and equation (65), it follows that $\varphi_c > 0$ if

$$F_c(Y^R|S) - F_c(Z(C^R)|S) < 0. \quad (66)$$

We now proceed to show that the inequality in (66) is met when the inequality in (40) is met. That the inequality in (39) is met implies that the inequality in (42) that is reproduced below, is met.

$$Z(C^R) > Y^{FB}(c) > Z^0(c).$$

Now

$$Z^0(C) < Z(C^R) \Rightarrow \{\beta P_S - [1 - P_S]TP_B - C^R\} < 0. \quad (67)$$

From the inequality in (38) and from $Y^R > Z(C^R)$

$$Z^0(C) < Z(C^R) \Rightarrow f_c(Z(C^R)|S) < 0 \Rightarrow f_c(Y^R)|S) < 0. \quad (68)$$

The inequality in (39) implies that the area under the $f_c(Z|S)$ curve decreases in the range $Z > Z^0(C)$. So it follows from (68) that

$$F_c(Y^R|S) - F_c(Z(C^R)|S) < 0.$$

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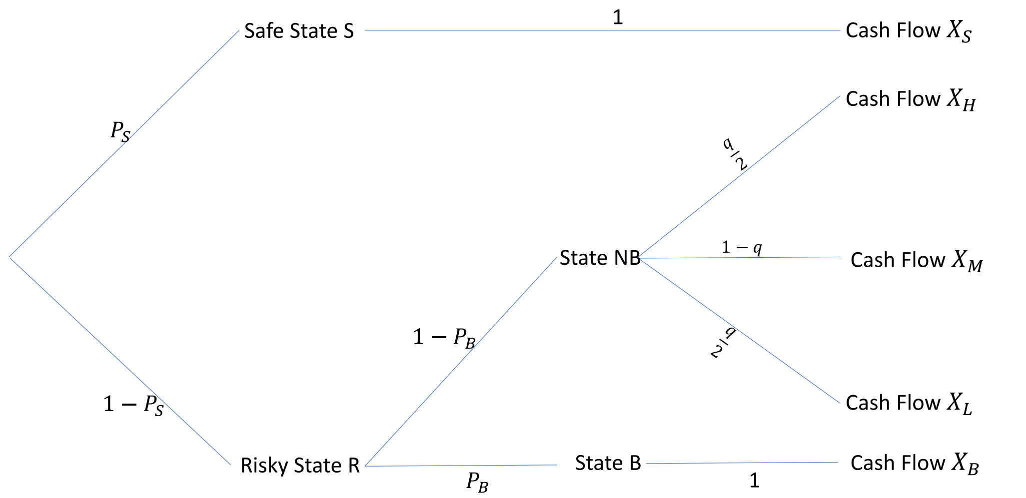
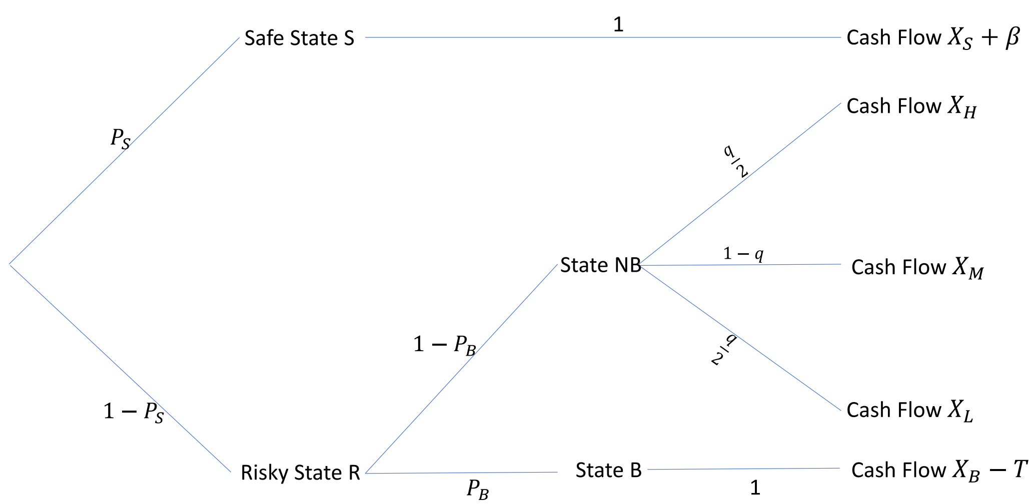
Figure 1: The Baseline Case of No Interim Investment and No Asset Substitution



$X_B < X_L < X_M \leq X_S < X_H$

Figure 2: The Case of Interim Investment without Asset Substitution



$X_B < X_L < X_M \leq X_S < X_H$

Figure 3: The Case of No Interim Investment with Asset Substitution

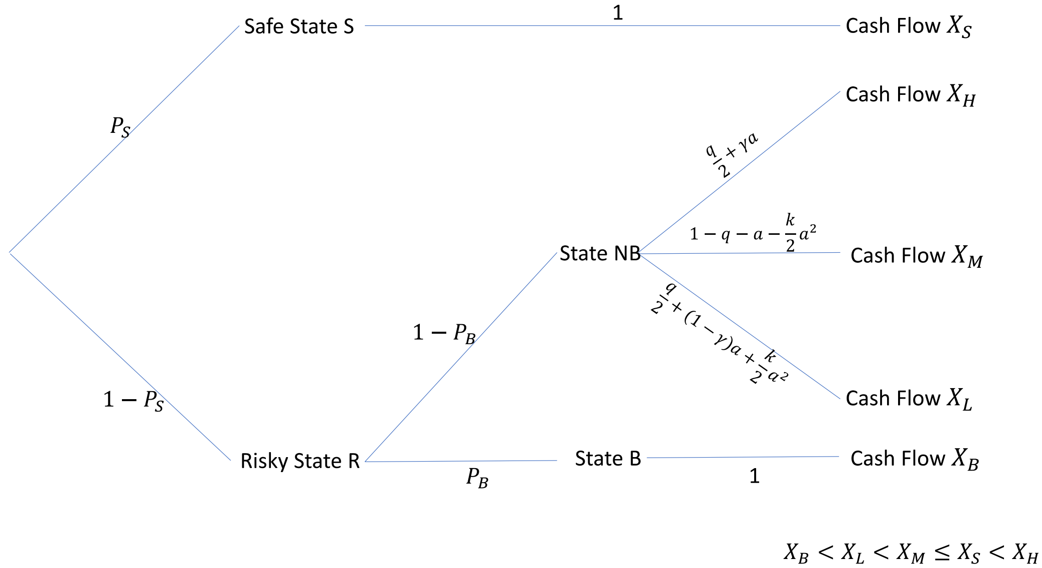


Figure 4: The Case of Interim Investment with Asset Substitution

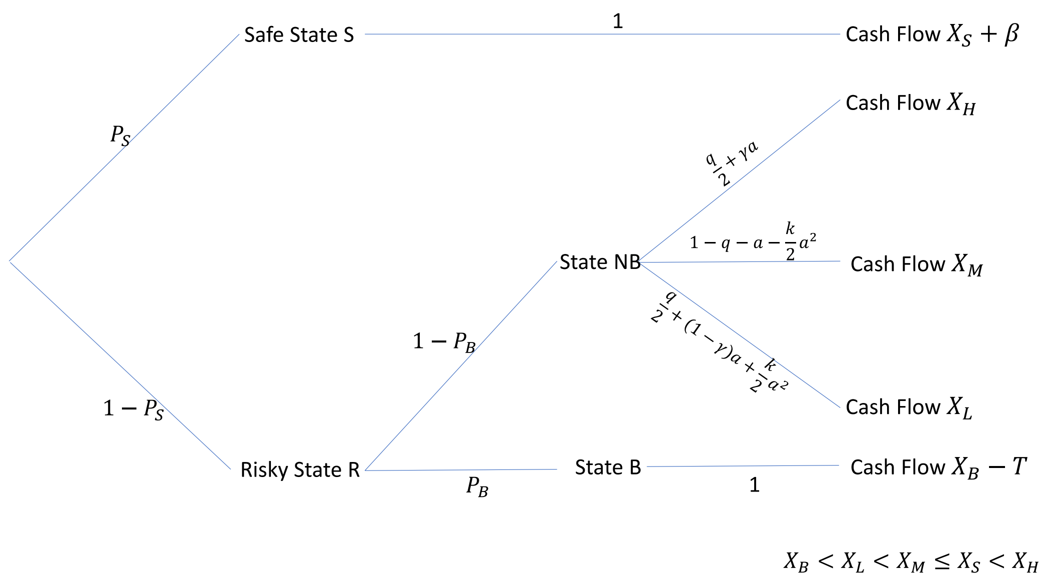


Figure 5: The Effect of Renegotiation Cost on the Optimal Covenant Tightness and the Renegotiation Region

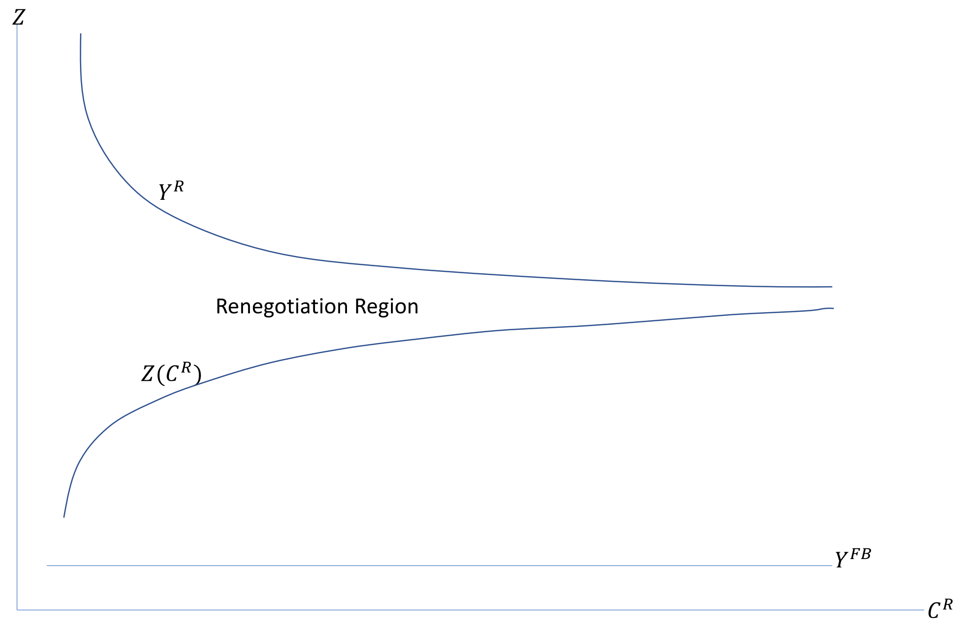


Figure 6: The Effect of Project Risk on the Optimal Covenant Tightness and the Renegotiation Region

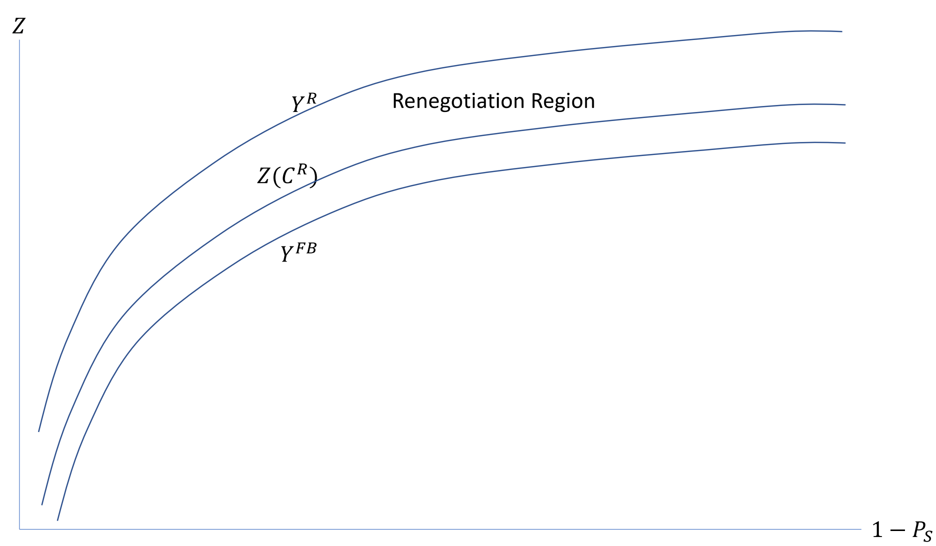


Figure 7: The Effect of Ease of Asset Substitution on the Optimal Covenant Tightness and the Renegotiation Region

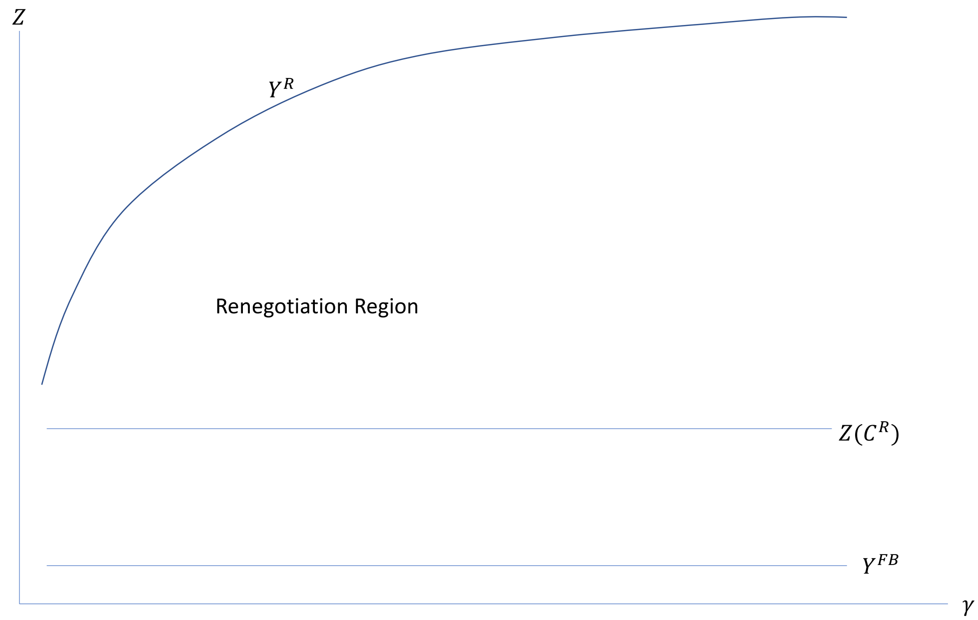


Figure 8: The Effect of Cost of Asset Substitution on the Optimal Covenant Tightness and the Renegotiation Region

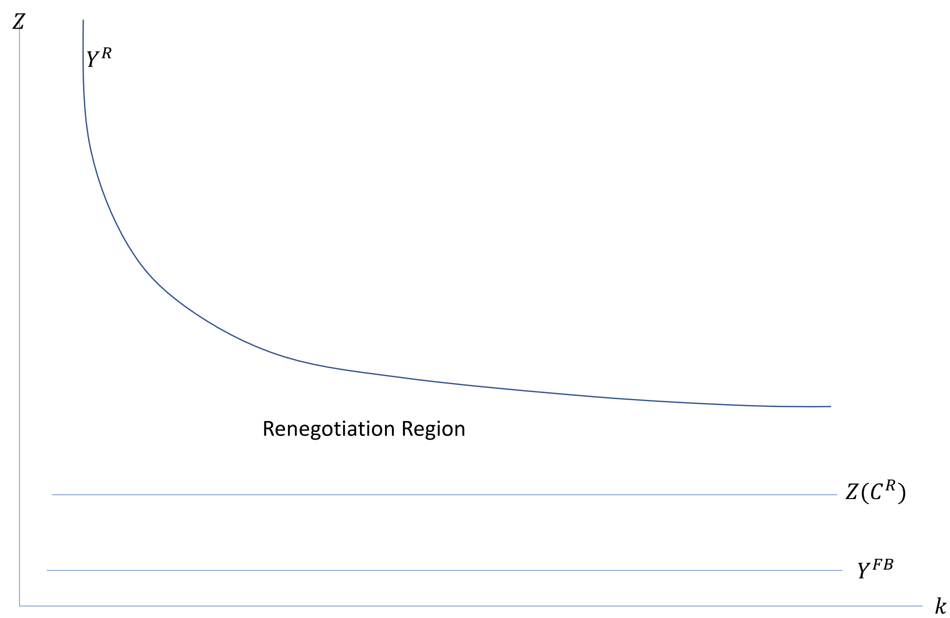


Figure 9: The Effect of Conservatism on the Optimal Covenant Tightness